

**Upper Tail Probabilites: v1.0:  
Probability and Quantile  
calculations for the  
HP48-GX**

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# 1 Introduction

Calculation of upper tail probabilities of conventional distributions is already included in the HP48G/GX. However, certain limitations are still encountered. First, inverses of the upper tail commands are not readily available. Short programs can be written that implement the inverses using numerical root finding techniques, such as the Newton Raphson method, the Secant method, or the Bisection method. In addition, the HP48G's ROOT command, which is a hybrid between the Secant method and other techniques could be used. This library makes use of the HP48G's ROOT command to implement the inverses of the upper tail commands. Second, the User's Guide mentions that the degrees of freedom associated with the upper tail commands must be between 0 and 499. Degrees of freedom larger than this result in a **Bad Argument Value** error. Using approximations taken from Abramowitz and Stegun's *Handbook of Mathematical Functions*, the functionality of the upper tail distributions are extended beyond 499 degrees of freedom. Third, all the built-in upper tail commands cannot be used in algebraic mode. All of the commands implemented in this library can be used in algebraic mode.

In addition, accurate calculation of upper tail probabilities of the Poisson, Binomial and Negative Binomial distribution was accomplished by noting relationships to the F and  $\chi^2$  distributions.

## 1.1 Disclaimer

UpperTail and its attached documentation are provided "as is", and are subject to change without prior notice. The author gives no warranty of any kind with regard to the software or documentation, either expressed or implied, including, but not limited to, the implied warranties of merchantability and fitness for a particular purpose. The author shall not be held liable for any damages, including any general, special, incidental, or consequential damages arising out of the use or inability to use any or all of the included programs.

Permission to copy the UpperTail library as a whole, unmodified package is granted provided that the copies are not made or distributed for resale (excepting nominal copying fees).

## 1.2 Installing and Deleting UpperTail

Size	3329.5 bytes
Checksum	#9594h [#38292d]
Version	1.0
Library Number	422

You can check these numbers by putting the library on the stack and pressing LS-[VAR] [BYTES]. The above statistics correspond to the distributed version, and if they do not match your results, your copy of the library may have been modified.

UpperTail should only work in a G/GX Calculator. No testing has been done other than Revision R (mine). However, as always, you should **BACKUP YOUR MEMORY** before running this application.

To install UpperTail:

1. Download the file `upper422.lib` into your calculator in binary mode.
2. Put the content of the created variable `upper422.lib` on the stack.
3. Delete the variable `upper422.lib`.
4. Store it in the port of your choice (for example with 0 STO.)

5. Power-cycle the calculator (OFF ON or ON-C).

To delete UpperTail:

1. Detach the library with `{:0:422} DETACH` (1,2 for the cards).
2. Purge the library with `{:0:422} PURGE` (1,2 for the cards). If you get an error message 'Object in use' then do ON-C and start again from 3.

The source code is given in the files `upper422src` or `upper422src.bz`. The `upper422src` file is an HP48 directory object, and the `upper422src.bz` is the compressed version of `upper422src` (compressed by Mika Heiskanen's BZ). Both of these are binary files and must be transferred to the calculator to be viewed. Compilation was done using Jazz v6.7.

### 1.3 UpperTail homepage

The UpperTail homepage is at <http://www.math.montana.edu/~hyde/>. The page always contains a link to the latest published version of UpperTail.

## 2 Distributions

### 2.1 Discrete Distributions

#### 2.1.1 The Binomial Distribution

The probability distribution for a binomial random variable,  $Y$ , is given by

$$p(y) = \Pr(Y = y) = \binom{n}{y} p^y (1-p)^{n-y} \quad (y = 0, 1, 2, \dots, n),$$

where

$$\begin{aligned} p &= \text{Probability of a success on a single trial,} \\ n &= \text{Number of Trials,} \\ y &= \text{Number of successes in } n \text{ trials,} \\ \text{and } \binom{n}{y} &= \frac{n!}{y!(n-y)!}. \end{aligned}$$

Further,

$$\Pr(Y > k) = \sum_{y=k+1}^n \Pr(Y = y) = \sum_{y=k+1}^n \binom{n}{y} p^y (1-p)^{n-y}.$$


---

#### 2.1.2 The Negative Binomial Distribution

The probability distribution for a negative binomial random variable,  $Y$ , is given by

$$p(y) = \Pr(Y = y) = \binom{y+r-1}{y} p^r (1-p)^y, \quad (y = 0, 1, 2, \dots),$$

where

$p$  = Probability of a success on a single trial,  
 and  $y$  = Number of failures until the  $r^{\text{th}}$  success.

Further,

$$\Pr(Y > k) = \sum_{y=k+1}^{\infty} \Pr(Y = y) = \sum_{y=k+1}^{\infty} \binom{y+r-1}{y} p^r (1-p)^y.$$


---

### 2.1.3 The Poisson Distribution

The probability distribution for a Poisson random variable,  $Y$ , is given by

$$p(y) = \Pr(Y = y) = \frac{\lambda^y e^{-\lambda}}{y!}, \quad (y = 0, 1, 2, \dots),$$

where

$\lambda$  = Mean number of events during a given unit of time, area, or volume,  
 and  $e$  = The base of the natural logarithm (2.718281828459...).

Further,

$$\Pr(Y > k) = \sum_{y=k+1}^{\infty} \Pr(Y = y) = \sum_{y=k+1}^{\infty} \frac{\lambda^y e^{-\lambda}}{y!}.$$


---

### 2.1.4 The Hypergeometric Distribution

The probability distribution for a hypergeometric random variable,  $Y$ , is given by

$$p(y) = \Pr(Y = y) = \frac{\binom{s}{y} \binom{N-s}{m-y}}{\binom{N}{m}}, \quad (y = \max[0, m - (N - s)], \dots, \min[s, m]),$$

where

$N$  = Total number of elements,  
 $s$  = Number with desired trait in the  $N$  elements,  
 $m$  = Number of elements drawn,  
 and  $y$  = Number with desired trait in the  $m$  elements drawn.

---

## 2.2 Continuous Distributions

### 2.2.1 The Normal Distribution

The Normal probability distribution has density given by

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}},$$

where

$\mu$  = mean,  
 and  $\sigma^2$  = variance ( $\sigma^2 > 0$ ).

Further,

$$\Pr(X > x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_x^\infty e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$


---

### 2.2.2 The $\chi^2$ Distribution

Let  $Y \sim \chi_\nu^2$ . The  $\chi^2$  probability distribution has density given by

$$f(y) = \frac{1}{2^{\frac{\nu}{2}} \Gamma\left(\frac{\nu}{2}\right)} y^{\frac{\nu}{2}-1} e^{-\frac{y}{2}},$$

where

$$\begin{aligned} y &> 0, \\ \nu &= \text{number of degrees of freedom,} \\ \text{and } \Gamma(x) &= \text{Euler's Gamma function.} \end{aligned}$$

Further,

$$\Pr(Y > y) = \frac{1}{2^{\frac{\nu}{2}} \Gamma\left(\frac{\nu}{2}\right)} \int_y^\infty t^{\frac{\nu}{2}-1} e^{-\frac{t}{2}} dt$$


---

### 2.2.3 Snedecor's F Distribution

Let  $Y \sim F(\nu_1, \nu_2)$ . The F probability distribution has density given by

$$f(y) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} y^{\frac{\nu_1}{2}-1} \left[1 + \frac{\nu_1}{\nu_2} y\right]^{-\frac{\nu_1+\nu_2}{2}},$$

where

$$\begin{aligned} y &> 0, \\ \nu_1 &= \text{numerator degrees of freedom,} \\ \nu_2 &= \text{denominator degrees of freedom,} \\ \text{and } \Gamma(x) &= \text{Euler's Gamma function.} \end{aligned}$$

Further,

$$\Pr(F > f) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \int_f^\infty x^{\frac{\nu_1}{2}-1} \left[1 + \frac{\nu_1}{\nu_2} x\right]^{-\frac{\nu_1+\nu_2}{2}} dx$$


---

### 2.2.4 Student's T Distribution

Let  $Y \sim T(\nu)$ . The T probability distribution has density given by

$$f(y) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\nu\pi}} \left(1 + \frac{y^2}{\nu}\right)^{-\frac{\nu+1}{2}},$$

where  $\nu$  = degrees of freedom,  
and  $\Gamma(x)$  = Euler's Gamma function.

Further,

$$\Pr(T < t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\nu\pi}} \int_t^\infty \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}} dx$$


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### 3 Quantile Commands

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#### 3.1 BINV

**Binomial Distribution Quantile Command:** Calculates the quantile associated with the given upper probability for the Binomial Distribution. This is the inverse of the function UPB.

Level 3	Level 2	Level 1	→	Level 1
$n$	$p$	$\alpha$	→	$k$

**Affected by:** UPF

**Remarks:** For  $Y \sim \text{Bin}(n, p)$ , BINV solves for  $k$  in the equation

$$\Pr(Y > k) \leq \alpha.$$

Explicitly,

$$\Pr(Y > k) = \Pr(F > f),$$

where  $F$  is Snedecor's F random variable with

$$\begin{aligned} j &= \text{FLOOR}(k) + 1, \\ \nu_1 &= 2(n - j + 1), \\ \nu_2 &= 2j, \end{aligned}$$

and

$$f = \frac{2j(1-p)}{2p(n-j+1)}.$$

**Related Commands:** UPB

---

#### 3.2 CINV

**$\chi^2$  Distribution Quantile Command:** Calculates the quantile associated with the given upper probability for the  $\chi^2$  distribution. This is the inverse of the function UPC.

Level 2	Level 1	→	Level 1
$\nu$	$\alpha$	→	$y$

**Affected by:** UPC

**Remarks:** Calculates the inverse of the UPC function, such that  $\text{CINV}(\nu, \alpha) = y$ .

**Related Commands:** UPC

---

### 3.3 FINV

**Snedecor's F Distribution Quantile Command:** Calculates the quantile associated with the given upper probability for Snedecor's F distribution. This is the inverse of the function UPF.

Level 3	Level 2	Level 1	→	Level 1
$\nu_1$	$\nu_2$	$\alpha$	→	$y$

**Affected by:** UPF

**Remarks:** Calculates the inverse of the UPF function, such that  $\text{FINV}(\nu_1, \nu_2, \alpha) = y$ .

**Related Commands:** UPF

---

### 3.4 NBINV

**Negative Binomial Distribution Quantile Command:** Calculates the quantile associated with the given upper probability for the Negative Binomial distribution. This version of the Negative Binomial distribution counts the number of failures before the  $r^{\text{th}}$  success. This is the inverse of the function UPNB.

Level 3	Level 2	Level 1	→	Level 1
$r$	$p$	$\alpha$	→	$k$

**Affected by:** UPF

**Remarks:** For  $Y \sim NB(r, p)$ , NBINV solves for  $k$  in the equation

$$\Pr(Y > k) \leq \alpha.$$

Explicitly,

$$\Pr(Y > k) = \Pr(F > f),$$

where  $F$  is Snedecor's F random variable with

$$\begin{aligned} j &= \text{FLOOR}(k) + 1, \\ \nu_1 &= 2r, \\ \nu_2 &= 2j, \end{aligned}$$

and

$$f = \frac{2jp}{2r(1-p)}.$$

**Related Commands:** UPNB

---

### 3.5 NINV

**Normal Distribution Quantile Command:** Calculates the quantile associated with the given upper probability for the Normal distribution. This is the inverse of the function UPN.

Level 3	Level 2	Level 1	→	Level 1
$\mu$	$\sigma^2$	$\alpha$	→	$y$

**Affected by:** UPN

**Remarks:** Calculates the inverse of the UPN function, such that  $\text{NINV}(\mu, \sigma^2, \alpha) = y$ .

**Related Commands:** UPN

---

### 3.6 PINV

**Poisson Distribution Quantile Command:** Calculates the quantile associated with the given upper probability for the Poisson distribution. This is the inverse of the function UPP.

Level 2	Level 1	→	Level 1
$\lambda$	$\alpha$	→	$k$

**Affected by:** UPC

**Remarks:** For  $Y \sim \text{POI}(\lambda)$ , PINV solves for  $k$  in the equation

$$\Pr(Y > k) \leq \alpha.$$

Explicitly,

$$\Pr(Y > k) = 1 - \Pr(X > x),$$

where  $X$  is a  $\chi^2$  random variable with

$$\begin{aligned} j &= \text{FLOOR}(k) + 1, \\ \nu &= 2j, \end{aligned}$$

and

$$x = 2\lambda.$$

**Related Commands:** UPP

---

### 3.7 TINV

**Student's T Distribution Quantile Command:** Calculates the quantile associated with the given upper probability for Student's T distribution. This is the inverse of the function UPT.

Level 2	Level 1	→	Level 1
$\nu$	$\alpha$	→	$y$

**Affected by:** UPT

**Remarks:** Calculates the inverse of the UPT function, such that  $\text{TINV}(\nu, \alpha) = y$ .

**Related Commands:** UPT

---

### 3.8 ZINV

**Standard Normal Distribution Quantile Command:** Calculates the quantile associated with the given upper probability for the standard Normal distribution  $N(0, 1)$ . This is the inverse of the function UPZ.

Level 1	→	Level 1
$\alpha$	→	$z$

**Affected by:** None

**Remarks:** Calculates the inverse of the UPZ function, such that  $ZINV(\alpha) = z$ .

**Related Commands:** UPZ, NINV

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## 4 Upper Tail Probability Commands

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### 4.1 HYPER

**Hypergeometric Distribution Command:** Produces a matrix giving the probabilities associated with the Hypergeometric distribution.

Level 3	Level 2	Level 1	→	Level 1
$N$	$s$	$m$	→	[ matrix ]

**Affected by:** COMB

**Remarks:** Produces a matrix of all possible outcomes of a Hypergeometric( $N, s, m$ ) distribution and associated probabilities. Launches the matrix into the built in Matrixwriter.

---

### 4.2 UPB

**Upper Binomial Distribution Command:**

Level 3	Level 2	Level 1	→	Level 1
$n$	$p$	$k$	→	$\alpha$

**Affected by:** UPF

**Remarks:** For  $Y \sim Bin(n, p)$ , UPB finds

$$\Pr(Y > k).$$

Explicitly,

$$\Pr(Y > k) = \Pr(F > f),$$

where  $F$  is Snedecor's F random variable with

$$\begin{aligned} j &= \text{FLOOR}(k) + 1, \\ \nu_1 &= 2(n - j + 1), \\ \nu_2 &= 2j, \end{aligned}$$

and

$$f = \frac{2j(1-p)}{2p(n-j+1)}.$$

**Related Commands:** BINV

---

### 4.3 UPC

**Upper  $\chi^2$  Distribution Command:** Returns the probability  $\text{UPC}(\nu, x)$  that a  $\chi^2$  random variable is greater than  $x$ , where  $\nu$  is the degrees of freedom of the distribution.

Level 2	Level 1	→	Level 1
$\nu$	$y$	→	$\text{UPC}(\nu, y)$

**Affected by:** None

**Remarks:** Let  $Y \sim \chi_\nu^2$ . For all positive  $y$ ,

$$\text{UPC}(\nu, y) = \Pr(Y > y) = \frac{1}{2^{\frac{\nu}{2}} \Gamma(\frac{\nu}{2})} \int_y^\infty t^{\frac{\nu}{2}-1} e^{-\frac{t}{2}} dt$$

For  $\nu > 499$ ,  $X$  is approximated by a standard Normal random variable. So

$$\Pr(Y > y) \approx \Pr(Z > z),$$

where

$$z = \frac{\sqrt[3]{\frac{y}{\nu}} - \left(1 - \frac{2}{9\nu}\right)}{\sqrt{\frac{2}{9\nu}}}.$$

**Related Commands:** CINV

---

#### 4.4 UPF

**Upper Snedecor's F Distribution Command:** Returns the probability  $\text{UPF}(\nu_1, \nu_2, f)$  that a Snedecor's F random variable is greater than  $f$ , where  $\nu_1$  and  $\nu_2$  are the numerator and denominator degrees of freedom of the F distribution.

Level 2	Level 1	→	Level 1
$\nu_1$	$\nu_2$	→	$\text{UPF}(\nu_1, \nu_2, f)$

**Affected by:** None

**Remarks:** Let  $F \sim F(\nu_1, \nu_2)$ . For all positive  $f$ ,

$$\text{UPF}(\nu_1, \nu_2, f) = \Pr(F > f) = \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \int_f^\infty x^{\frac{\nu_1}{2}-1} \left[1 + \frac{\nu_1}{\nu_2} x\right]^{-\frac{\nu_1 + \nu_2}{2}} dx$$

For  $\nu_1 > 499$  or  $\nu_2 > 499$ ,  $F$  is approximated by a standard Normal random variable. So

$$\Pr(F > f) \approx \Pr(Z > z),$$

where

$$z = \frac{\sqrt[3]{f} \left(1 - \frac{2}{9\nu_2}\right) - \left(1 - \frac{2}{9\nu_1}\right)}{\sqrt{\frac{2}{9\nu_1} + \sqrt[3]{f^2} \frac{2}{9\nu_2}}}.$$

**Related Commands:** FINV

---

#### 4.5 UPN

**Upper Normal Distribution Command:** Returns the upper tail probability associated with the  $N(\mu, \sigma^2)$  distribution, where  $\mu$  and  $\sigma^2$  are the mean and variance, respectively.

Level 3	Level 2	Level 1	→	Level 1
$\mu$	$\sigma^2$	$x$	→	$\text{UTP}(\mu, \sigma^2, x)$

**Affected by:** None

**Remarks:** Let  $X \sim N(\mu, \sigma^2)$ . For all  $x$ ,  $\mu$ , and positive  $\sigma^2$ ,

$$UPN(\mu, \sigma^2, x) = \Pr(X > x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_x^\infty e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt.$$

For  $\sigma^2 = 0$ , UPN returns 0 for  $x \geq \mu$ , and 1 for  $x < \mu$ .

**Related Commands:** NINV

---

## 4.6 UPNB

**Upper Negative Binomial Distribution Command:**

Level 3	Level 2	Level 1	→	Level 1
$r$	$p$	$k$	→	$\alpha$

**Affected by:** UPF

**Remarks:** For  $Y \sim NB(r, p)$ , UPNB calculates

$$\Pr(Y > k).$$

Explicitly,

$$\Pr(Y > k) = \Pr(F > f),$$

where  $F$  is Snedecor's F random variable with

$$\begin{aligned} j &= \text{FLOOR}(k) + 1, \\ \nu_1 &= 2r, \\ \nu_2 &= 2j, \end{aligned}$$

and

$$f = \frac{2jp}{2r(1-p)}.$$

**Related Commands:** NBINV

---

## 4.7 UPP

**Upper Poisson Distribution Command:**

Level 2	Level 1	→	Level 1
$\lambda$	$k$	→	$\alpha$

**Affected by:** UPC

**Remarks:** For  $Y \sim POI(\lambda)$ , UPP calculates

$$\Pr(Y > k).$$

Explicitly,

$$\Pr(Y > k) = 1 - \Pr(X > x),$$

where  $X$  is a  $\chi^2$  random variable with

$$\begin{aligned} j &= \text{FLOOR}(k) + 1, \\ \nu &= 2j, \end{aligned}$$

and

$$x = 2\lambda.$$

**Related Commands:** PINV

---

## 4.8 UPT

**Upper Student's T Distribution Command:** Returns the probability  $\text{UPT}(\nu, t)$  that a Student's T random variable is greater than  $t$ , where  $\nu$  is the degrees of freedom of the distribution.

Level 2	Level 1	→	Level 1
$\nu$	$t$	→	$\text{UPT}(\nu, t)$

**Affected by:** None

**Remarks:** Let  $T \sim T_\nu$ . For all  $t$ ,

$$\text{UPT}(\nu, t) = \Pr(T < t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}} \int_t^\infty \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}} dx.$$

For  $\nu > 499$ ,  $T$  is approximated by a standard Normal random variable. So

$$\Pr(T > t) \approx \Pr(Z > z),$$

where

$$z = \frac{t\left(1 - \frac{1}{4\nu}\right)}{\sqrt{1 + \frac{t^2}{2\nu}}}.$$

**Related Commands:** TINV

---

## 4.9 UPZ

**Upper Standard Normal Distribution Command:** Returns the upper tail probability associated with the standard Normal distribution  $N(0, 1)$ .

Level 1	→	Level 1
$z$	→	$\text{UPZ}(z)$

**Affected by:** None

**Remarks:** Let  $Z \sim N(0, 1)$ . For all  $z$ ,

$$\text{UPZ}(z) = \Pr(Z > z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-\frac{1}{2}t^2} dt.$$

**Related Commands:** UPN, ZINV

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## 5 Miscellaneous

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### 5.1 HELP422

**Help Command:** This command displays stack syntax for the commands of this library

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