



hp calculators

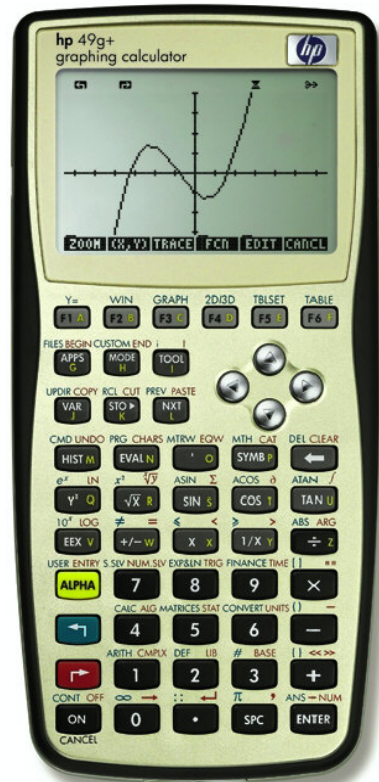
HP 49G+ Symbolic integration of polynomials

Methods used

The integration commands

The substitution commands

Practice integrating polynomials symbolically



Methods used

The HP49G+ provides large selection of methods for performing symbolic integration and for finding antiderivatives. Several methods for the symbolic integration of expressions involving polynomials are considered here.

Integration commands

The provided integration commands are INT, INTVX, RISCH and \int . Any of these commands can be used for symbolic integration in combination with substitution, expansion, and so on. The command INT is accessible using the built-in command catalogue of the HP49G+. Press \rightarrow CAT to open the catalogue. From the catalogue you can select and execute any of the existing commands. The catalogue is much like a menu of an application, where you can use the arrow keys to select menu items, or jump to the items typing the first few letters of them. While the catalogue is active, press ALPHA ALPHA T N T to jump to the command INT. Pressing the key ENTER or the menu key MENU will put the selected item on the command line (or execute the selected item if RPN mode is on). Pressing EXIT will quit the command catalogue without executing the selected item. The command INT needs three arguments: The expression to be integrated, the variable of integration, and the value of the variable of integration where the antiderivative will be evaluated.

The commands INTVX and RISCH are available in the menu "Derivatives and Integrals" This menu is accessed pressing \leftarrow CALC to open the "Calculus" menu.



Figure 1

The first menu item is 1.DERIV & INTEG.... and it is highlighted (selected). In this CHOOSE box selecting 1.DERIV & INTEG... and pressing ENTER or MENU takes you to a new menu which contains differentiation and integration commands:



Figure 2

The commands INTVX and RISCH are in the second page of the menu, so you must press 7 to have the CHOOSE box scroll down and see them. The command INTVX is provided as a shorter way to perform integrations as it only needs one argument, the expression to be integrated, and uses automatically the current CAS variable VX (usually X) as the variable of integration. RISCH needs two arguments: the expression to be integrated and the variable of integration.

Finally, the command \int is accessible from the keyboard pressing \rightarrow \int . It needs four arguments: the lower and upper limit of integration, the expression that must be integrated, and the variable of integration. In many cases, this will be the command that is the best choice for numeric integration.

The substitution commands

The commands for substitution are SUBST, | (where), and PREVAL. The command PREVAL allows for the substitution and evaluation of the difference $g(x_2)-g(x_1)$, where $g(x)$ is the antiderivative of some function $f(x)$ that we want to integrate between the limits x_1 and x_2 . This command resides in the menu 1.DERIV & INTEG.... The command SUBST allows for the substitution of the variable of integration, since it will take care of altered integration limits and other necessary substitutions in the integral. This command resides in (the second page) of the menu "Algebra" which you access by pressing \rightarrow ALG .

Practice integrating polynomials symbolically

Example 1: Find the antiderivative of the polynomial: $X^4 - 5X^3 + 18X^2 - 60X + 120$

Solution: Assume algebraic exact mode, CHOOSE boxes, and X as the current CAS variable VX. Start the equation writer and call the calculus menu.

 EQW  CALC



Figure 3

ENTER (Choose the menu 1.DERIV & INTEG....)




Figure 4

8 (Select the command INTVX)



Figure 5

 (Put the command INTVX with its place holders in the equation writer)

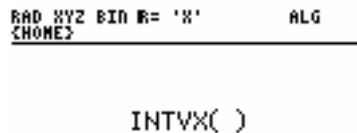


Figure 6

$x^4 - 5x^3 + 3x^2 + 1$ (Enter the polynomial)

and perform the integration.

ENTER ENTER

```

RAD XYZ BIN R= 'X'      ALG
<HOME>

:INTVX(X^4-5X^3+18X^2-60X
1/5X^5-51/4X^4+181/3X^3-601/2X
x  k  S3  A1  S2  S1

```

Figure 7

Answer: $\frac{1}{5}x^5 - 5\frac{1}{4}x^4 + 18\frac{1}{3}x^3 - 60\frac{1}{2}x^2 + 120x$

Note that since the antiderivative of a function is only determined up to an additive constant, the above result is only one of the possible antiderivatives. The general result is:

$$\frac{1}{5}x^5 - 5\frac{1}{4}x^4 + 18\frac{1}{3}x^3 - 60\frac{1}{2}x^2 + 120x + C$$

where C is the additive constant.

Example 2: Find the antiderivative of:

```

RAD XYZ BIN R= 'X'
<HOME>
4:
3:
2:
1:
x^2+1
x^2-1
x  k  S3  A1  S2  S1

```

Figure 8

Solution: Assume RPN exact mode, CHOOSE boxes and X as current variable VX.

$\left(\rightarrow\right) \text{EQW} \left(X\right) \left(Y^x\right) \left(2\right) \left(\rightarrow\right) \left(+\right) \left(/ \right) \left(\rightarrow\right) \left(\wedge\right) \left(\div\right) \left(X\right) \left(Y^x\right) \left(2\right) \left(\rightarrow\right) \left(-\right) \left(/ \right) \left(\text{ENTER}\right)$

```

RAD XYZ BIN R= 'X'
<HOME>
4:
3:
2:
1:
x^2+1
x^2-1
x  k  S3  A1  S2  S1

```

Figure 9

$\left(\leftarrow\right) \text{CALC} \left(\text{ENTER}\right) \left(8\right) \left(\text{ENTER}\right)$

```

RAD XYZ BIN R= 'X'
<HOME>
7:
6:
5:
4:
3:
2:
1: X+LN(X+1)+LN(X-1)
x  k  S3  A1  S2  S1

```

Figure 10

$\left(\rightarrow\right) \text{ALG} \left(4\right) \left(\text{ENTER}\right)$ (Collect the logarithms)

RAD XYZ BIN R= 'X'
{HOME}
5:
4:
3:
2:
1:
$$x - \ln\left(\frac{x+1}{x-1}\right)$$

x k S3 A1 S2 S1

Figure 11

Answer:

Example 3: Integrate symbolically for Y

RAD XYZ BIN R= 'X'
{HOME}
4:
3:
2:
1:
$$\sqrt{y^2+2}$$

x k S3 A1 S2 S1

Figure 12

Solution: Assume RPN exact mode with CHOOSE boxes on.

Enter the expression using the EquationWriter.

\rightarrow EQW \sqrt{x} ALPHA Y y^x 2 \rightarrow \div ALPHA Y y^x 2 \rightarrow + 2 ENTER

RAD XYZ BIN R= 'X'
{HOME}
4:
3:
2:
1:
$$\sqrt{y^2+2}$$

x k S3 A1 S2 S1

Figure 13

\rightarrow ALPHA Y ENTER \leftarrow CALC ENTER \downarrow ENTER (to find the antiderivative – this takes a few seconds).

RAD XYZ BIN R= 'X'
{HOME}
4:
3:
2:
1:
$$\text{INT}\left(\frac{\sqrt{y^2+2}}{\sqrt{y^2+2}}, y, Y\right)$$

x k S3 A1 S2 S1

Figure 14

\rightarrow ALG 2 ENTER

RAD XYZ BIN R= 'X'
{HOME}
5:
4:
3:
2:
1:
$$\frac{\sqrt{y^2+2}}{y}$$

x k S3 A1 S2 S1

Figure 15

Answer:

Example 4: Integrate symbolically:

RAD XYZ BIN R= 'X'
{HOME}

4:
3:
2:
1:

$$\frac{x^2-1+(x^2+1)\sqrt{1-x^2}}{x^4-1}$$

x	R	S3	R1	S2	S1
---	---	----	----	----	----

Figure 16

Solution: Assume RPN mode with soft menus and X as the current variable VX. Enter the expression on the stack.

EQW 2 2
 2 4 ENTER

```

RAD HY2 BID R= 'X'
(CHOME)
4:
3:
2:
1:

$$\frac{x^2-1+(x^2+1)\sqrt{1-x^2}}{x^4-1}$$


```

Figure 17

\leftarrow CALC ENTER 8 ENTER (solve the integral)

```

RAD XYZ BIN R= 'X'
#HOME#
7:
6:
5:
4:
3:
2:
1:      ATAN(X)+ASIN(X)-

```

Figure 18

Answer:

Example 5: One physical body moves with $v_x=3t+1$ and $v_y=2$. Find its location vector as a function of time.

Solution: Since $x = \int v(t) dt$ we have to integrate the above velocities v_x and v_y . Assume algebraic mode CHOOSE boxes. Build-up the command.

(←) CALC (ENTER) (÷) (↓) (ENTER) (←) (I) (3) (×) (ALPHA) (←) (I) (+) (÷) (→) (—) (2)

```

BAD WY2 BID B= 'X'      ALG
CHOME)

```

```

RISCH([3*t+1,2],t)

```

y	k	c2	a1	c2	c1

Figure 19

ENTER

```

RAD XYZ BIN R= 'X'      ALG
{HOME}

:RISCH([3*t+1 2],t)
      [3*1/2*t^2+t 2*t]
x  k  S3  A1  S2  S1

```

Answer:

Figure 20