



hp calculators

HP 49G+ Symbolic Differentiation

Methods used

The differentiation commands

Practice solving symbolic differentiation problems



Methods used

To perform symbolic differentiation we just use one of the differentiation commands. This is rather straightforward since the 49G+ built-in CAS can find symbolic derivatives of all built-in functions in any combination.

The differentiation commands

The HP49G+ provides three basic commands for differentiation which we can use for finding numeric values of derivatives. The simplest provided commands for differentiation are: DERIV, DERVX and ∂ . The command ∂ is available from the keyboard by pressing the keys $\rightarrow \partial$. The other two commands are available in several menus. For example, to display the calculus menu, press $\leftarrow \text{CALC}$.



Figure 1

Its first menu item is 1.DERIV & INTEG.... and if ENTER is pressed, a menu which contains differentiation and integration commands is displayed.



Figure 2

The menu items 2.DERIV and 3.DERVX are the other two basic differentiation commands. Any of these three commands can be used for symbolic differentiation. The commands DERIV and ∂ both take two arguments: The expression to be differentiated and the variable with respect to which we want to differentiate. DERVX is provided as a shorter way to perform differentiation when the variable of differentiation is the same with the CAS variable VX (usually X). The command DERIV will also accept a scalar expression and a vector of names, or even a vector of algebraic objects and a vector of names as arguments, allowing the user to find gradients or even hessian matrices. There are also other commands related to differentiation in this menu, like CURL, DIV, or HESS. The command CURL takes a three dimensional vector field and a three dimensional variables vector and returns the rotational of the field. The command DIV takes a vector field and a vector of variables and return the divergence of the field. The command HESS takes a multivariate function and a vector of variables and returns the hessian matrix, the gradient and the used vector of variables.

Practice solving symbolic differentiation problems

Example 1: Find the derivative of the function $X^2 e^{-X}$

Solution: $\rightarrow \text{EQW} \rightarrow \partial \rightarrow X \rightarrow X \rightarrow Y^X \rightarrow 2 \rightarrow X \rightarrow \leftarrow e^X \rightarrow X \rightarrow +/- \rightarrow \text{ENTER}$

$$\frac{d}{dx}(x^2 \cdot e^{-x})$$

Figure 3

EVAL

$$\frac{e^X \cdot 2X - X^2 \cdot e^X}{SQ(e^X)}$$

Figure 4

→ ALG 2 ENTER

$$\frac{-X^2 - 2X}{e^X}$$

Figure 5

Answer:Example 2: Find the derivative of the function $X^2 \sin(X)$ Solution:

→ EQW → d/dx X → X Y^X 2 → X SIN X ENTER

$$\frac{d}{dx}(x^2 \cdot \sin(X))$$

Figure 6

EVAL

$$2X \cdot \sin(X) + X^2 \cdot \cos(X)$$

Figure 7

Answer:Example 3: Find the derivative of the function:

$$\frac{\partial}{\partial Y} \left(\frac{\partial}{\partial X} \left(\frac{X^2 + Y^2}{X \cdot Y} \right) \right)$$

Solution:
 $\boxed{\rightarrow} \boxed{EQW} \boxed{\rightarrow} \boxed{\partial} \boxed{X} \boxed{\rightarrow} \boxed{X} \boxed{Y^x} \boxed{2} \boxed{\rightarrow} \boxed{+} \boxed{ALPHA} \boxed{Y} \boxed{Y^x} \boxed{2} \boxed{\rightarrow} \boxed{\div} \boxed{X} \boxed{X} \boxed{ALPHA} \boxed{Y} \boxed{ENTER}$

$$\frac{\partial}{\partial X} \left(\frac{X^2 + Y^2}{X \cdot Y} \right)$$

Figure 8

 $\boxed{\nabla} \boxed{\rightarrow} \boxed{\partial} \boxed{ALPHA} \boxed{Y} \boxed{ENTER}$

$$\frac{\partial}{\partial Y} \left(\frac{\partial}{\partial X} \left(\frac{X^2 + Y^2}{X \cdot Y} \right) \right)$$

Figure 9

 $\boxed{EVAL} \boxed{\rightarrow} \boxed{ALG} \boxed{2} \boxed{ENTER}$

$$-\frac{X^2 + Y^2}{Y^2 \cdot X^2}$$

Figure 10

Answer:Example 4: Find the gradient of the function $x^2y - y^2x$ Solution:
 $\boxed{\rightarrow} \boxed{EQW} \boxed{X} \boxed{Y^x} \boxed{2} \boxed{\rightarrow} \boxed{X} \boxed{ALPHA} \boxed{Y} \boxed{-} \boxed{ALPHA} \boxed{Y} \boxed{Y^x} \boxed{2} \boxed{\rightarrow} \boxed{X} \boxed{X} \boxed{ENTER}$

$$X^2 \cdot Y - Y^2 \cdot X$$

Figure 11

 Now input the vector of variables onto the stack. $\boxed{\leftarrow} \boxed{[]} \boxed{X} \boxed{Y} \boxed{ENTER}$

$$X^2 \cdot Y - Y^2 \cdot X$$

Figure 12

\leftarrow CALC \leftarrow ENTER 2 \leftarrow ENTER (Note: this finds the gradient)

```

RAD XYZ BIN R= 'X'
{HOME}
6:
5:
4:
3:
2:
1: [Y^2.X-Y^2.X^2-X^2.Y]
x k S3 A1 S2 S1

```

Figure 13

\leftarrow ALG 2 \leftarrow ENTER

```

RAD XYZ BIN R= 'X'
{HOME}
6:
5:
4:
3:
2:
1: [2.Y.X-Y^2.X^2-2.Y.X]
x k S3 A1 S2 S1

```

Figure 14

Answer:

Example 5: Differentiate the vector: $\left[\frac{1}{Y \cdot Z} \frac{1}{X \cdot Z} \frac{1}{X \cdot Y} \right]$ for Z. Assume Algebraic mode.

Solution:

Create the vector: \leftarrow $\left(\frac{1}{Y \cdot Z} \right)$ $\left(\frac{1}{X \cdot Z} \right)$ $\left(\frac{1}{X \cdot Y} \right)$ \leftarrow ALPHA $\left(\frac{1}{Z \cdot Y} \right)$ $\left(\frac{1}{Z \cdot X} \right)$ $\left(\frac{1}{Y \cdot X} \right)$ \leftarrow ENTER

```

RAD XYZ BIN R= 'X'      ALG
{HOME}
6:
5:
4:
3:
2:
1: [1/Y.Z 1/X.Z 1/X.Y]
x k S3 A1 S2 S1

```

Figure 15

\leftarrow CALC \leftarrow ENTER 2 \leftarrow ENTER \leftarrow ANS \leftarrow ENTER , ALPHA $\left(\frac{1}{Z} \right)$

```

RAD XYZ BIN R= 'X'      ALG
{HOME}
6:
5:
4:
3:
2:
1: [1/Y.Z 1/X.Z 1/X.Y]
x k S3 A1 S2 S1

```

Figure 16

\leftarrow ENTER

```

RAD XYZ BIN R= 'X'      ALG
{HOME}
6:
5:
4:
3:
2:
1: [1/Y.Z 1/X.Z 1/X.Y]
x k S3 A1 S2 S1

```

Figure 17

\rightarrow $\overline{\text{ALG}}$ $\boxed{2}$ $\overline{\text{ENTER}}$ \leftarrow $\overline{\text{ANS}}$ $\overline{\text{ENTER}}$

```

RAD XYZ BIN R= 'X'      ALG
{HOME}
:DERIV(ANS(1),Z)
      
$$\left[ \frac{-(Y \cdot 1)}{\text{SQ}(Z \cdot Y)} \quad \frac{-(X \cdot 1)}{\text{SQ}(Z \cdot X)} \quad 0 \right]$$

:EXPAND(ANS(1))
      
$$\left[ \frac{-1}{Z^2 \cdot Y} \quad \frac{-1}{Z^2 \cdot X} \quad 0 \right]$$


```

x	k	S3	R1	S2	S1
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Figure 18

Answer: