hp calculators

HP 49G+ Symbolic integration of trigonometric functions

Methods used

The integration commands

The substitution commands

Practice solving symbolic integration problems involving trigonometric functions

Methods used

The HP49G+ provides large selection of methods for performing symbolic integration and for finding antiderivatives. Several methods for the symbolic integration of expressions involving trigonometric functions are considered here. This training aid only scratches the surface of what the 49G+ can do.

Integration commands

The provided integration commands are INT, INTVX, RISCH and ∫. Any of these commands can be used for symbolic integration in combination with substitution, expansion, and so on. The command INT is accessible using the built-in command catalogue of the HP49G+. Press ↱ CAT to open the catalogue. From the catalogue you can select and execute any of the existing commands. The catalogue is much like a menu of an application, where you can use the arrow keys to select menu items, or jump to the items typing the first few letters of them. While the catalogue is active, press ALPHA ALPHA I N T to jump to the command INT. Pressing the key ENTER or the menu key OK will put the selected item on the command line (or execute the selected item if RPN mode is on). Pressing CANCEL will quit the command catalogue without executing the selected item. The command INT needs three arguments: The expression to be integrated, the variable of integration, and the value of the variable of integration where the antiderivative will be evaluated.

The commands INTVX and RISCH are available in the menu "Derivatives and Integrals" This menu is accessed pressing ↰ CALC to open the "Calculus" menu.

Figure 1

The first menu item is 1.DERIV & INTEG.... and it is highlighted (selected). In this CHOOSE box selecting 1.DERIV & INTEG... and pressing ENTER or OK takes you to a new menu which contains differentiation and integration commands:

Figure 2

The commands INTVX and RISCH are in the second page of the menu, so you must press 7 to have the CHOOSE box scroll down and see them. The command INTVX is provided as a shorter way to perform integrations as it only needs one argument, the expression to be integrated, and uses automatically the current CAS variable VX (usually X) as the variable of integration. RISCH needs two arguments: the expression to be integrated and the variable of integration.

Finally, the command ∫ is accessible from the keyboard pressing ↱ ∫ . It needs four arguments: the lower and upper limit of integration, the expression that must be integrated, and the variable of integration. In many cases, this will be the command that is the best choice for numeric integration.

The substitution commands

The commands for substitution are SUBST, | (where), and PREVAL. The command PREVAL allows for the substitution and evaluation of the difference $g(x_2)-g(x_1)$, where g(x) is the antiderivative of some function f(x) that we want to integrate between the limits x_1 and x_2. This command resides in the menu 1.DERIV & INTEG..... The command SUBST allows for the substitution of the variable of integration, since it will take care of altered integration limits and other necessary substitutions in the integral. This command resides in (the second page) of the menu "Algebra" which you access by pressing ↱ ALG .

Practice solving symbolic integration problems involving trigonometric functions

Example 1: Find the antiderivative of the function

$$\text{SIN}(X)\cdot\text{COS}\left(\frac{X}{2}\right)$$

Solution: Assume algebraic exact mode and CHOOSE boxes.

[→] EQW [←] CALC

Figure 3

ENTER (Choose the menu 1.DERIV & INTEG.....)

Figure 4

1 1 (Press the key 1 twice to jump to the command RISCH)

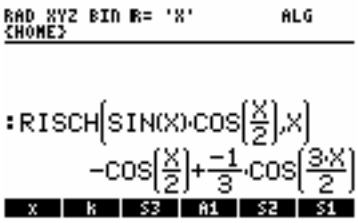
Figure 5

ENTER (Put the command RISCH with its place holders in the equation writer)

Figure 6

Enter the arguments for RISCH and perform integration.

SIN X ▲ ▲ × COS X ÷ 2 ▶ X ENTER ENTER



Answer: Figure 7

Note that since the antiderivative of a function is only determined up to an additive constant, the above result is only one of the possible antiderivatives. The general result is:

$$-\cos\left(\frac{X}{2}\right)+\frac{-1}{3}\cdot\cos\left(\frac{3\cdot X}{2}\right)+C$$

where C is the additive constant.

Example 2: Find the antiderivative of:

$$\frac{SIN(X)-COS(X)}{SIN(X)+COS(X)}$$

Solution: Assume RPN exact mode, CHOOSE boxes and X as current variable VX. Enter the expression,

→ EQW SIN X ▶ − COS X → ▲ ÷ SIN X ▶ + COS X

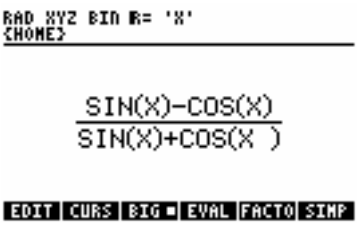


Figure 8

ENTER

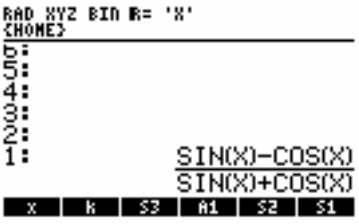
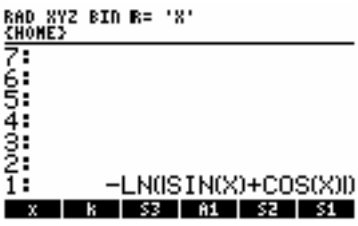


Figure 9

← CALC ENTER 8 ENTER (Perform the integration)



Answer: Figure 10

Example 3: Integrate symbolically

$$\mathrm{TAN}\left(3\cdot X-\frac{\pi}{3}\right)$$

Solution: Assume RPN exact mode. Put the expression on stack level 1.

→ EQW TAN 3 × X − ← π ÷ 3 ENTER

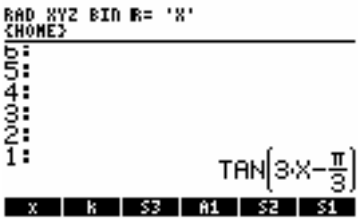


Figure 11

Enter variable X.

' X ENTER

We use X as the value at which the antiderivative will be evaluated, so press ENTER to duplicate the variable:

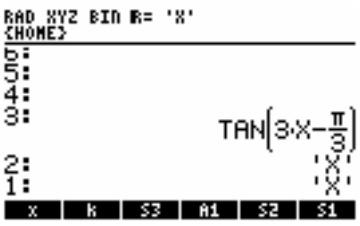
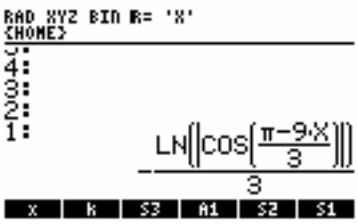


Figure 12

ALPHA ALPHA I N T ENTER (Issue the command INT – note how you can type commands this way)



Answer: Figure 13

Example 4: Until now we examined integration problems that the HP49G+ is able to solve without any user intervention. But there are also integration problems that need some manipulation by the user to allow the HP49G+ to solve them. Using the great variety of commands that the HP49G+ provides, we can rewrite some expression in such a way that the subsequent integration will be successful. For example:

Integrate symbolically: SIN(LN(X))

Solution: Assume RPN mode with CHOOSE boxes and X as the current variable VX. Enter the integral.

→ EQW ALPHA ALPHA I N T ALPHA ← ( ) SIN → LN X ▶ X ▶ X ENTER

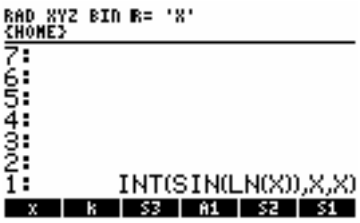


Figure 14

Attempt integration.

↱ ALG 2 ENTER

The HP49G+ returns the same integral unsolved with variable X substituted by Xtt:

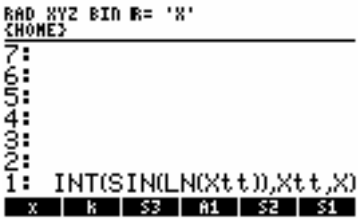


Figure 15

But the integral is solvable on the HP 49G+. Substitute LN(x)=y in the original integral.

↱ UNDO ↱ EQW ↱ LN X ▶ ↱ = ALPHA Y ENTER

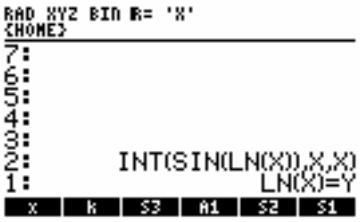


Figure 16

↱ ALG 8 ENTER (Perform the substitution)

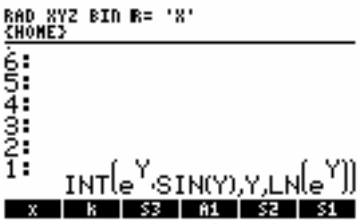


Figure 17

Notice that the HP49G+ didn't only replace the expression LN(x) by y. It also transformed the variable of integration X to e^Y and changed the evaluation point of the integral to $LN(e^Y)$.

↱ ALG 2 ENTER

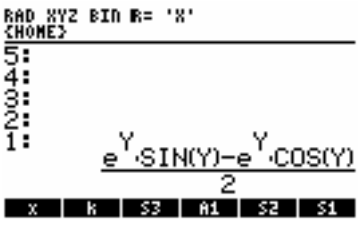


Figure 18

→ EQW ALPHA Y → = → LN X ENTER → ALG 8 ENTER (Substitute back Y=LN(X))
→ ALG 2 ENTER (Display the correct result).

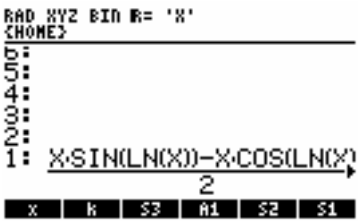


Figure 19

Answer: $\frac{X\cdot SIN(LN(X))-X\cdot COS(LN(X))}{2}$

Example 5: Integrate symbolically $e^{ACOS(X)}$

Solution: Assume algebraic mode with CHOOSE boxes. Attempt integration.

→ EQW ALPHA ALPHA I N T ALPHA ◀ ( ) ◀ e^x ◀ ACOS X ▶ X ▶ X ENTER ENTER

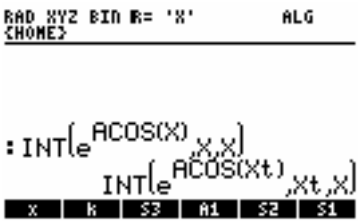


Figure 20

→ ALG 8 ENTER ◀ ANS → , ◀ ACOS X ALPHA ◀ T ▶ → = ALPHA Y ENTER

Figure 21

Notice again that the HP49G+ replaced the expression arccos(Xt) by y and also performed all necessary transformations.

→ ALG 2 ◀ ANS ENTER

Figure 22

Answer: $\frac{\left(x-\sqrt{-\left(x^2-1\right)}\right)\cdot e^{\mathrm{ACOS}(x)}}{2}$

Example 6: Find the symbolic result for the integral:

$$\int\int \mathrm{SIN}(X+Y)^2\,dX\,dY$$

Solution: Assume RPN mode and CHOOSE boxes.

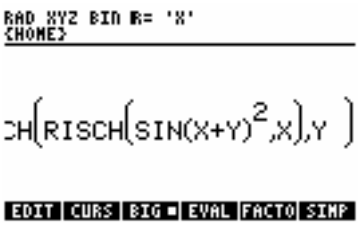
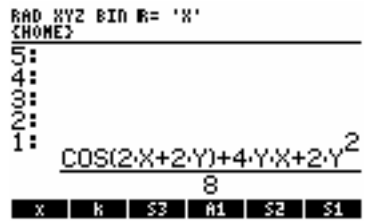


Figure 23

ENTER → ALG 2 ENTER (expand and solve the integral)



Answer:

Figure 24