USING THE DESOLVE COMMAND

Here are some results I found while working with ODEs. Did I stay up all night creating equations that would come out nicely? Well, no ─ I swiped them from a textbook. **[1]**

The DESOLVE command is for solving differential equations. Despite the CAT Help statement implying that it only does "some first-order equations", it's actually much more versatile.

SETTING UP:
─ The calculator should be in Real mode, Exact mode, and radian mode.
─ Flag 3 should be clear for symbolic mode.
─ To do this all at once, run **CASCFG**.

Also, numbers should be integers ─ see **Using Initial Conditions** below.

Note that running DESOLVE creates a variable called ODETYPE in the current dir. It contains a string showing the type of ODE you just solved, such as "1st order linear". The usefulness of this isn't obvious.

DESOLVE also writes the working variable into VX - it appears in the header. CASCFG will reset it to the default 'X'.

STARTING:
Run CASCFG (CAT or type in)

  Solve:

$$\frac{dy}{dx} + 2 \cdot xy = 5 \cdot x$$

 In DESOLVE syntax, dy/dx (or y') is written as **d1Y(X)** (or d1y(x) ─ you can use lower case if you want). The equation goes on Level 2 and Level 1 gets the variable of interest ─ **'Y(X)'**.

You can also use any other variables if you keep to the format ─ *eg*, the textbook dA/dB is the same as the DESOLVE d1A(B).

DESOLVE integrates both sides of the equation, where y is the variable we want and x becomes the variable of integration. For instance, in solving y' = x, DESOLVE performs

$$\int \frac{dy}{dx} = \int x \, dx$$

The textbook result would be

$$y = \frac{x^2}{2} + C.$$

You don't have to put the equation in any particular order — DESOLVE takes care of the details.

Finally, when you're entering an equation from a book, all occurrences of Y (or y) should appear as Y(X) (or y(x)). Y by itself will be taken as a separate variable and will cause errors.

The calculator format of the example:

```
'd1Y(X)+2*X*Y(X)=5*X'
          'Y(X)'
```

DESOLVE will return `{'Y(X)=(5*EXP(X^2)/2+cC0)/(1/EXP(X^2))'}`

There are several things to note: it's in a list, it's an equation of the form 'y(x)=expression', and it contains **cC0**.

The list can be removed by pressing EVAL. A second EVAL will attempt to simplify (SIMPLIFY has the same effect). This gives

```
'Y(X)=(5*EXP(X^2)+2*cC0)/(2*EXP(X^2))'
```

The cC0 is the arbitrary constant, usually just C in textbooks. DESOLVE is the only one of the integrators to insert it for you. If there's more than one, they'll be in the order cC0, cC1, etc.

The various simplifying commands all leave the expression as shown, more or less. However, if you edit out the Y(X)=, `ALG-PARTFRAC` will return

$$5/2 + \frac{cC0}{e^{X^2}}$$

The textbook answer is $5/2 + C * e^{-X^2}$. Close enough.

If it won't simplify, you can try Approximate mode, though it may give large reals that make the expression hard to read. Point of interest: when you're setting up the equation, you can move terms around all you want, and spaces are allowed. For example, the equation could be:

```
'd1Y(X) + 2*X*Y(X) - 5*X = 0'
                        'Y(X)'
```

Spaces will be removed on the stack. The result is the same.

USEFUL TOOLS

If you plan on using DESOLVE a lot, it's worth assigning the command to a key. I use the 7 key, which is 72.1.

```
<< DESOLVE >>  72.1  ASN
```

I also have a var called **ODEFORM**, which contains

```
{ 'd1Y(X)=0'  'Y(X)' }
```

It's easy to edit that into a new equation. Manipulating a result is easier if you remove the Y(X)= that DESOLVE uses in answers. This program does that:

```
<< EVAL EQ→ SWAP DROP >>
```

It's stored as **SIMPDE**. It removes the list and the Y(X)=. You can add another EVAL or SIMPLIFY at the end for simplifying if you want, but it's usually worth checking the full answer first. I've used SIMPDE in the examples below ─ if you don't use it, it means manually editing to some extent.

A second point of interest: if you put the answer into the EQW with the down-arrow cursor, you can highlight certain sections of it and press SIMP. It might be faster than a manual edit.

GETTING STUCK:

This will happen. The calculator's integration is not perfect, and some integrals have no closed form. An example of the latter is in the HP manuals:

```
'd1y(x)+x^2*y(x)=5'
```

DESOLVE returns { 'y(x) =(5*INT(EXP(xt^3/3),xt,x)+cC0)*1/EXP(x^3/3)}' }

The **INT** and the **xt** tell you that the calculator isn't going to take it any further because e^x^3/3 has no closed form. The HP manuals have more to say about converting this to a form that can be solved numerically.

There's also the problem that the answer may not look even faintly like the textbook result. As the textbook authors point out, "A differential equation can have infinitely many particular solutions." This can mean infinitely more trouble for you. :-)

USING INITIAL CONDITIONS

A nice feature of DESOLVE is the ability to accept initial conditions along with the equation. The equation and conditions are enclosed in an array. Numbers should be integers (convert any decimals first with `CONVERT-REWRITE-NXT- →Q)`.

Solve

$$y - x\frac{dy}{dx} + 1 = 4x^4$$

for the initial condition that y=1/3 when x=1.

The initial conditions will be written `'Y(1)=1/3'`.

    ['Y(X)-X*d1Y(X)+1=4*X^4' 'Y(1)=1/3'] 'Y(X)'

DESOLVE returns `{'Y(X)=(-(4/3*X^3--1/X+8/3)*X'`

SIMPDE returns `'(-(4/3*X^3--1/X)+8/3)*X'`

EVAL produces `-((4*X^4-8*X+3)/3)'`

The **Where** command (the vertical line, RS-TOOL) makes it easy to solve for any value of X, say, 1.5:

    -((4*X^4-8*X+3)/3)' {X 1.5} | →NUM   → -3.75

Or you can use `'X=1.5'` SUBST for the same result.

SEPARATED DIFFERENTIALS

DE equations are often given in the form

$$(2y - 6x^2)dx + xdy = 0$$

There's no direct way to enter this into DESOLVE, but you can rewrite it to get dy/dx by dividing through by dx:

$$2y - 6x^2 + x\frac{dy}{dx} = 0$$

In DESOLVE: `'X*d1Y(X)+2*Y(X)-6*X^2=0'`
                              `'Y(X)'`

This gives `'(3/2*X^4+cC0)*(1/X^2)'` after SIMPDE.

Simplifying commands return only `'(2*cC0+3*X^4)/(2*X^2)'`, but PARTFRAC comes to the rescue again, returning `3/2*X^2 + cC0/X^2`,

which agrees with the textbook $\dfrac{3}{2}X^2 + \dfrac{C}{X^2}$

A SECOND-ORDER EQUATION
DESOLVE will solve second-degree (and higher) equations. A simple example:

 Solve y" - 3y' = 0, also written as

$$\frac{d^2 y}{dx^2} - 3\frac{dy}{dx} = 0.$$

In DESOLVE, the second derivative, y", becomes `d1d1Y(X)`.

```
'd1d1Y(X)-3*d1Y(X)=0'
            'Y(X)'
```

After simplifying, the result is:

```
'(cC1*EXP(3*X)+(3*cC0-cC1))/3'
```

$\dfrac{C2\,e^{3X} + 3\,C1 - C2}{3}$ is the textbook version.

AN APP: NEWTON'S LAW OF COOLING
In the 1600s, Isaac Newton pointed out (along with everything else worth knowing, apparently [2]) that an object cools at a rate proportional to the difference between the object's temperature and the ambient temperature.

A thermometer reading 75 degrees F is placed in an ambient temperature of 40 degrees. After 5 minutes, the reading is 60 degrees. Find the reading after 7 minutes.

The equation is

$$\frac{dT}{dt} = k(T - Ta)$$

where
    T = object temp
    Ta = ambient temp, in this case always 40
    t = time
    k is a constant allowing for conditions and must be determined by measurements

The by-hand solution for this has a lot of steps, but DESOLVE's initial conditions feature shortens it somewhat. We know that T=75 when t=0, or T(0)=75, so that goes into the array after the equation:

```
['d1T(t)=k*(T(t)-40)'  'T(0)=75']
                          'T(t)'
```

This returns, after SIMPDE and EVAL:

```
'35*EXP(t*k)+40'
```
   (the value for T, the object temp)

Duplicate this with ENTER - we'll need it twice.

We know that T=60 when t=5, so we can find k. Enter 60 and press = to make an equation, then enter 't=5' and the stack should show:

```
        '35*EXP(t*k)+40'
    '35*EXP(t*k)+40=60'
                  't=5'
```

Run SUBST to replace the t with 5.

Enter 'k' and press S.SLV-SOLVE (or ISOL):

```
          '35*EXP(t*k)+40'
    'k=-(LN(7)-2*LN(2))/5'
```

The presence of k makes the simplifying commands refuse to get out of symbolic mode, so switch to **Approximate** mode (RS+ENTER) and press EVAL. The stack should show

```
        'EXP(t*k)+40'
    'k=-.111923157588'
```

We need to insert k into the expression on Lev 2, and also t=7 to get the final answer. SUBST can do both with an array **[3]**, so enter 't=7' (or press CMD to retrieve 't=5' and edit that) and then a **2**. Press PRG-TYPE- →ARRY. The stack should be:

```
              'EXP(t*k)+40'
    ['k=-.111923157588'  't=7']
```

Run SUBST and EVAL to get 55.9887597282.

If your thermometer can display trillionths of a degree, there you go. Otherwise, 56 degrees should do it.

ANOTHER APP: CHARGING A CAPACITOR

If you look in electronic handbooks for the charging of a capacitor through a resistor, you'll see this classic term:

$$e^{\frac{-t}{RC}}$$

It's usually in an equation that gives a value with respect to current or voltage. The demonstration circuit is something like the one below, left. The plot at the right shows the circuit current decreasing as the capacitor charges through R from a constant voltage DC when the switch is closed. The X-axis is time.
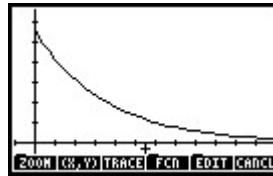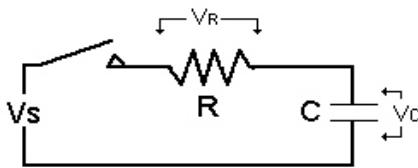
However, few books ever explain where this comes from. A dig through some EE texts produced some equations.

As Mr. Kirchoff pointed out, the voltages in the series circuit elements have to equal the supply voltage Vs, (or later, zero, which is what the constant Vs becomes after differentiating).

$$V_R + V_C = Vs$$

Since the current is the same in all parts of a series circuit, how it changes applies to both R and C, so we'd like to use it for our working variable. In the resistor, V=iR, so differentiating throughout returns

$$R\frac{d}{dt}i(t)+\frac{dVc(t)}{dt}=0$$

The second term is still a voltage and we'd like it in terms of current. The current in a capacitor is

$$i=C\frac{dV}{dt}$$

To put that in DESOLVE format, I've used upper-case **I** because of the ongoing struggle between the engineer's *i* (current) and the calculator's *i* ($\sqrt{-1}$).

```
'C*d1V(t)=I'
       'V(t)'
```

This returns `'V(t)=I*t/C+cC0'`. We can take cC0 as zero and ignore it, and since *I* varies with time *t*, we can take it as *I(t)*.

The capacitor current is now with respect to time, so we can directly substitute for the voltage derivative in our equation:

$$R\frac{d}{dt}i(t)+\frac{i(t)}{C}=0$$

We could put that straight into DESOLVE, but if we multiply through by $1/R$ to isolate the derivative, it starts to look more familiar:

$$\frac{d}{dt}i(t)+\frac{i(t)}{RC}=0$$

In DESOLVE format:

```
'd1I(t)+I(t)/(R*C)=0'
            'I(t)'
```

On the stack, in textbook display, this returns:

$$I(t)=cC0\cdot\frac{1}{e^{\frac{t}{RC}}}$$

Which is the same thing as:

$$I(t)=cC0\cdot e^{\frac{-t}{RC}}$$

The constant cC0 usually represents current, so our neglected *Vs* can return. When the switch is first closed, the capacitor is empty and equal to a short circuit, so for the first instant the current is just *Vs/R*. Our final expression for the current is:

$$i=\frac{Vs}{R}\cdot e^{\frac{-t}{RC}}$$

When **t** is very large with respect to **RC**, e^-t/RC can be taken as zero and current stops flowing. In fact, there's a rule of thumb from this: the capacitor can be considered charged at 5RC seconds (RC is called the time constant) . At 5RC, the current is down to 0.0067 times the maximum, so the capacitor voltage is about 99.3% of maximum, adequate for most purposes.

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[1] And a very good textbook it is, too. *Technical Calculus*, Dale Ewan *et al,* Pearson Education, Prentice Hall, 2002. There's a 2005 edition called *Technical Mathematics with Calculus*, though the calculus section is about the same.

[2] He also wrote millions of useless words on alchemy. But that's the way it was back then.

[3] Using an array with SUBST seems to be undocumented.