

## General

RotC, library nr 823, contains commands for the conversion of rotations between different forms of expression. Also 3 quaternion operators.

Input is accepted if values can be represented as real numbers. OK examples:

0.23 1  $\sqrt{3}/2$   $\pi/6$  COS(22)

RPN mode recommended. Exact or approximate modes.

Array input can be either user type 3 or 29.

Validity of input values are checked, with respect to:

Matrices are 3x3, orthonormal and righthanded.

Quaternions are 4 element vectors with unit length.

Output have real number elements.

## Command summary

->EUL	Matrix or quaternion to Euler angles
->RPY	Matrix or quaternion to RPY angles
EUL->Q	Euler angles to quaternion
EUL->M	Euler angles to matrix
RPY->Q	RPY angles to quaternion
RPY->M	RPY angles to matrix
M<-->Q	Toggles between matrix and quaternion
Q<-->θV	Toggles between quaternion and angle, axis of rotation
QxQ	2 successive quaternion rotations to resulting rotation
QxV	Quaternion rotation of vector
Q->Qc	Reverse of quaternion rotation
RCLIM	Plus version features

## Download contents

Two library versions are included, basic (file RC5basic.lib, ~8 kB) and plus (file RC5plus.lib, ~18 kB). In the plus version the user has the possibilities to:

- Select behaviour when a command is started with none, or wrong type of, arguments. The alternatives are launch of an online help, or raising an error.
- Change limiting values for acceptance of matrix and quaternion input.
- Allow quaternion output of any magnitude and with negative first element.

The basic version always errors on wrong arguments, and use default limits for checking input values. The default limits corresponds roughly to a precision of 5 decimal places. Quaternion output is by default of unit length. By default, first element of quaternion output will always be positive.

Also in the download are 2 optional and separate programs:

RotC. A choose box for access to the library commands. See installation section on page 8.

RCclean. It removes any of the 2 variables in the hidden directory which may have been created by the plus version. RCclean can be EVALuated directly from the SD card.

## Command reference

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### ->EUL

Factorizes a rotation, expressed as matrix or quaternion, to angles in Euler sequence. All 12 axis orders are valid. The axis order can be omitted, or specified either as a string or as a vector of integers.

Input, 3 alternatives:

<b>Level 2</b>		Matrix/quaternion	Matrix/quaternion
<b>Level 1</b>	Matrix/quaternion	"Axis order"	[ Axis order ]

If no axis order is specified the default order "ZYX" will be used.

Output is lists with the possible angle sequences. Angles are tagged with the axis id. Two sequence lists are returned when the input can be factorized to 2 unique sequences. The lists are ordered in increasing angular distance. When the input can be factorized to an infinite number of sequences, 3 lists are returned, where the third list involves parameter angle t.

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### ->RPY

As **->EUL**, except that factorization is to angles in RPY sequence.  
Default axis order is "XYZ".

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### EUL->Q

Converts an Euler sequence to a rotation expressed as quaternion. All 12 axis orders are valid. The axis order can be omitted, or specified either as a string or as a vector of integers.

Input, 3 alternatives:

<b>Level 4</b>		Angle 1st axis	Angle 1st axis
<b>Level 3</b>	Angle 1st axis	Angle 2nd axis	Angle 2nd axis
<b>Level 2</b>	Angle 2nd axis	Angle 3rd axis	Angle 3rd axis
<b>Level 1</b>	Angle 3rd axis	"Axis order"	[ Axis order ]

If no axis order is specified the default order "ZYX" will be used.

Output is a quaternion. Vector of user type 3.

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### EUL->M

As **EUL->Q**, except that output is a rotation matrix.  
Matrix is of user type 3.

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### RPY->Q

As **EUL->Q**, but for RPY type sequence. Default axis order is "XYZ".

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### RPY->M

As **EUL->M**, but for RPY type sequence. Default axis order is "XYZ".

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### M<-->Q

Converts a rotation expressed as matrix to quaternion, or vice verse.

Input/output, 2 alternatives:

**Level 1**     Matrix             Quaternion

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**Level 1**     Quaternion         Matrix

Output is of user type 3 or 29, same as input.

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### Q<-->θV

Converts a rotation expressed as quaternion to an angle and axis of rotation, or vice verse. Axis of rotation is a 3D vector. As input it can have any length. As output it has unit length.

Input/output, 3 alternatives:

**Level 2**                             Angle                     Axis of rotation

**Level 1**     Quaternion             Axis of rotation             Angle

->

**Level 2**     Angle

**Level 1**     Axis of rotation             Quaternion             Quaternion

Vector output is user type 3 or 29, same as array input.

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### QxQ

Returns the result of a sequence of two rotations, where the rotations are expressed as quaternions.

Input/output:

**Level 2**     1st rotation as quaternion

**Level 1**     2nd rotation as quaternion

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**Level 1**     The composite rotation as quaternion

Output is of user type 29 if both inputs are type 29. Otherwise user type 3.

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### QxV

Rotates a 3D vector, where the rotation is expressed as quaternion.

Input/output:

**Level 2**     Rotation as quaternion

**Level 1**     Vector to be rotated

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**Level 1**     Vector after rotation

Output is of user type 29 if both inputs are type 29. Otherwise user type 3.

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## Q->Qc

Quaternion conjugate. Returns the reverse of a rotation expressed as quaternion.

Input/output:

**Level 1**      Rotation as quaternion

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**Level 1**      Reverse rotation as quaternion.

Output is of user type 3 or 29, same as input.

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## RCLIM

In plus version only. For user settings.

If input cannot be processed, the default behaviour is to launch an online help. The help applies both to the current command as well as to general information. As an user selected alternative the help can be replaced by triggering an error in the standard way. If so, the variable RCerrOnWrongArg will be created in the hidden directory. This variable will be destroyed when default behaviour is restored.

Quaternion input is checked for normalization. Matrix input is checked for orthonormality and righthandedness. The limiting values for these checks can be changed by input forms in RCLIM. The default limiting values are:

Max quaternion magnitude	1.000005
Min quaternion magnitude	0.999995
Max magnitude of each matrix row	1.00001
Min magnitude of each matrix row	0.99999
Max deviation from right angle between matrix rows	0.0005 degrees
Matrices required to be righthanded (i.e. positive determinant).	

In addition, quaternion output will be of unit length and with positive scalar part (first element), unless changed by user.

The user can change limiting values, and also the form of quaternion output. User customizations are stored in a variable in the hidden directory. The variable name is RCLimValues. When defaults are restored this variable is destroyed.

Quaternion and matrix output is filtered with respect to element values being close to -1, 0 or 1. This filtering is suppressed if user has customized the settings.

To see the effects of filtering, in degrees mode: Convert the angle/axis pair 120 and [ 1 1 1 ] to quaternion and then to matrix, when defaults are active and not.

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## Help screens

From the plus version. By "frame" is meant an orthonormal coordinate system in three dimensions.

### Matrix

One way to represent a rotation is as a 3x3 matrix.

The matrix must be:  
orthonormal  
righthanded

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Matrix rows are base axes of a rotated frame, as viewed from the reference frame.

Columns are base axes of reference frame, as viewed from rotated frame.

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Matrix multiplication in reverse order gives the result of two successive rotations of a frame.

Rot A, then B gives C. A and C are relative global frame, B rel A.

Matrix mult:  $C = B * A$

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Matrix multiplication can also be applied to rotate a vector.

A = rotation, as 3x3 matrix.

V = vector to rotate, as 1x3 matrix.

W = rotated vector.

Matrix mult:  $W = V * A$

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### Quaternion

A quaternion is a 4D vector.

If normalized, it can be seen as containing an angle and axis of rotation. Angle and axis together takes a reference frame to the rotated frame.

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1st quaternion element is cosine of half the rotation angle.

Last 3 elements are axis of rotation, as a vector whose length is sine of half the rotation angle.

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1st element = scalar part

Last 3 elements = vector part

Scalar part =  $\cos(\theta/2)$

Vector part = Rot axis

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Quaternions can be multiplied to give the result of two successive rotations of a frame.

Rot p, then q gives r. p and r are relative global frame, q rel p.

Quat mult:  $r = p * q$

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Quat multiplication:

$p * q = r$

scalar part =  $p_0, q_0, r_0$

vector part =  $pV, qV, rV$

$r_0 = p_0 * q_0 - \text{DOT}(pV, qV)$

$rV = p_0 * qV + q_0 * pV + \text{CROSS}(pV, qV)$

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The conjugate of a quaternion has vector part negated.

The conjugate reflects a rotation in reverse.

Similar to transpose of a rotation matrix.

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Quat mult can be used to rotate a vector.

p = rotation as quat.

v = vector to rotate, as quat with 0 as scalar part.

p<sup>-</sup> = conjugate of p.

w = v rotated.

Quat mult:  $w = p * v * p^{-}$

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### Euler and RPY sequences

An Euler/RPY sequence of angles specifies 3 successive rotations, around base axes of a coordinate frame.

Result of sequence depends on direction and order of base axes

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Euler sequence:

Axes of rotation are base axes of the frame being rotated.

RPY (Roll, Pitch, Yaw):

Axes of rotation are static base axes of the reference frame.

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Euler/RPY angles can be related to 1 of 12 axis orders:

XYZ XZY YXZ ZXZ

YZX YXZ YZY YXY

ZXY ZYX ZXZ ZYZ

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Given one axis order, a rotation can be the result of 2 different Euler or RPY sequences

Exceptions occurs when mid angle is 0°, ±90° or 180°. Depends axis order. See next screen

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Infinitely many angle sequences give same result, if mid angle:

±90° if axis order is of XYZ .. ZYX type.

0°, 180° if axis order is of YXZ .. ZYZ type.

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Euler - RPY sequences give equal result if:  
1. The axis orders are reversed  
2. Axis/angles are same

Euler Z:30 Y:60 X:70

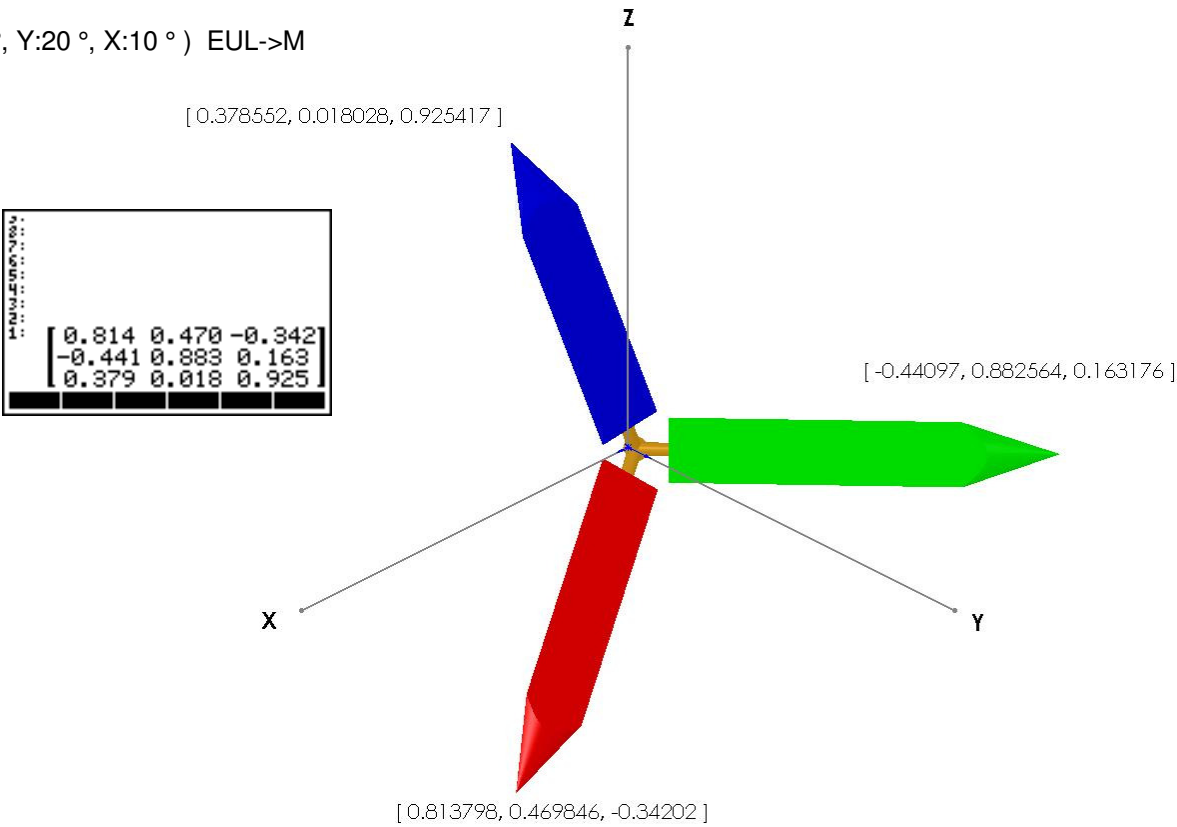
= RPY X:70 Y:60 Z:30

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Visualizations

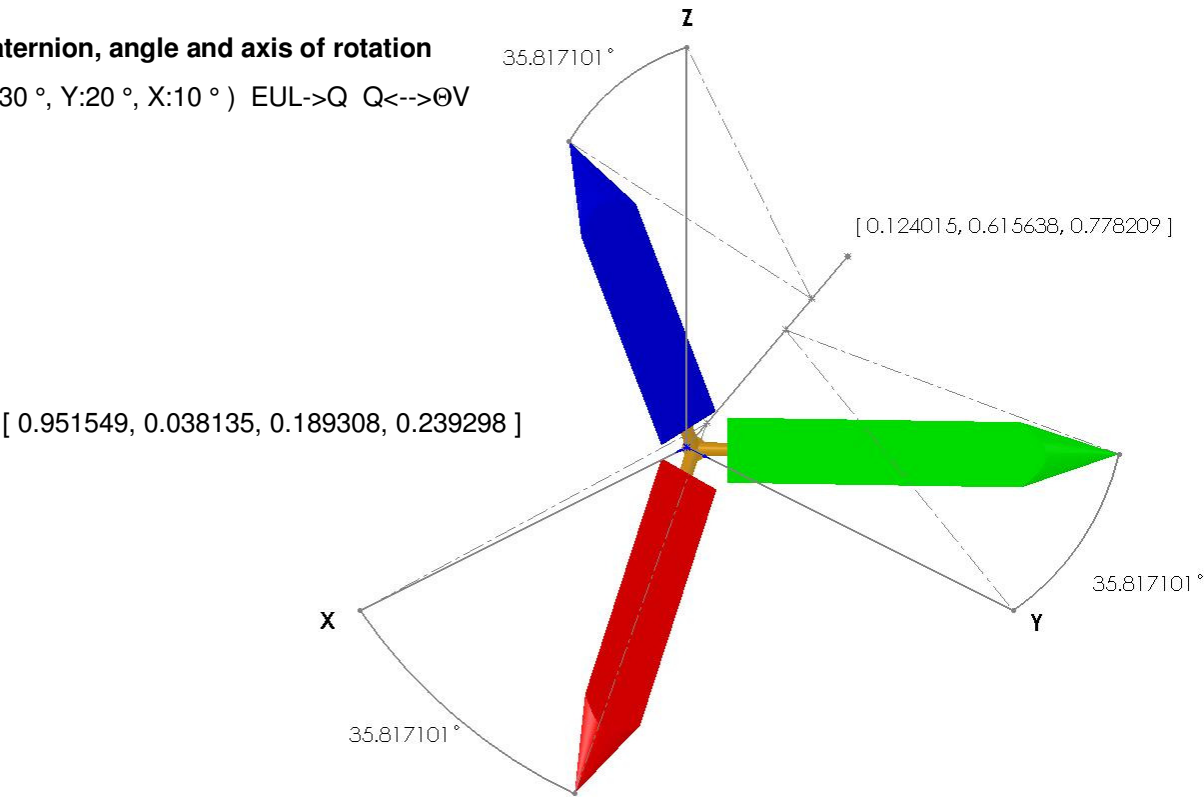
Matrix

( Z:30 °, Y:20 °, X:10 ° ) EUL->M



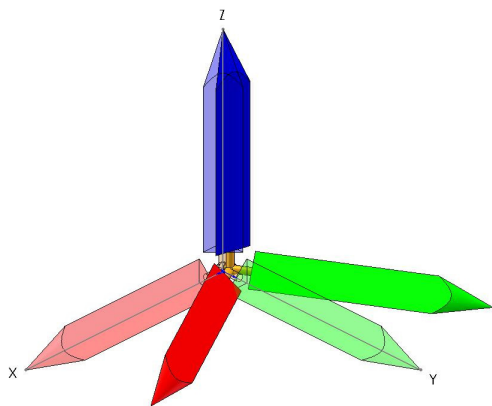
Quaternion, angle and axis of rotation

( Z:30 °, Y:20 °, X:10 ° ) EUL->Q Q<-->OV

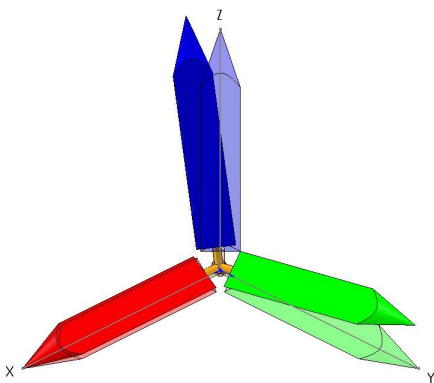


Euler and RPY sequences

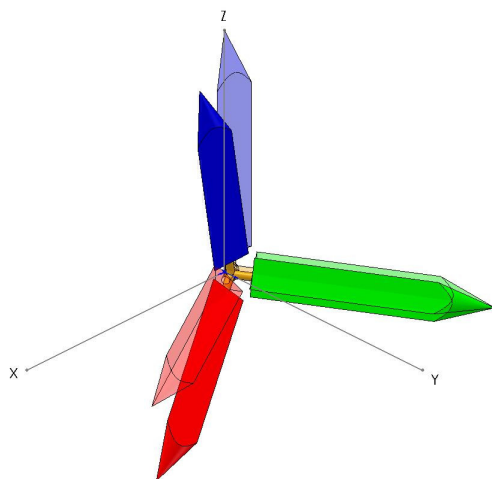
Euler Z 30 °



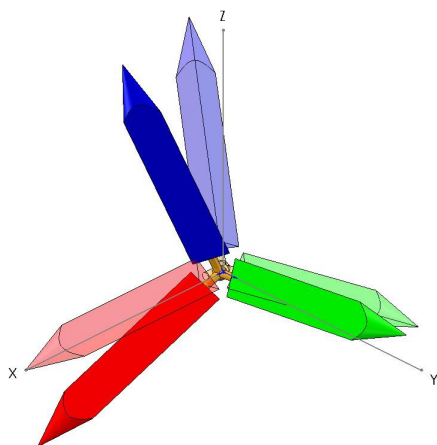
RPY X 10 °



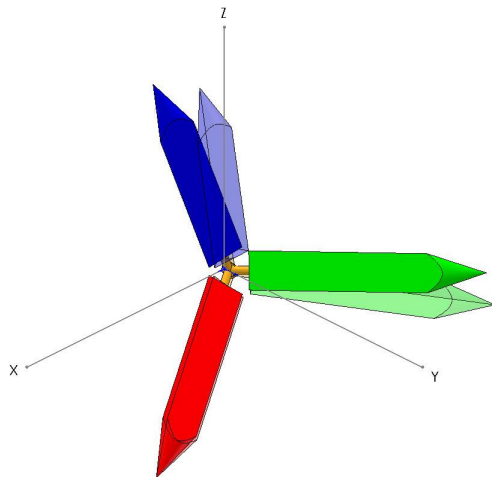
Euler Y 20 °



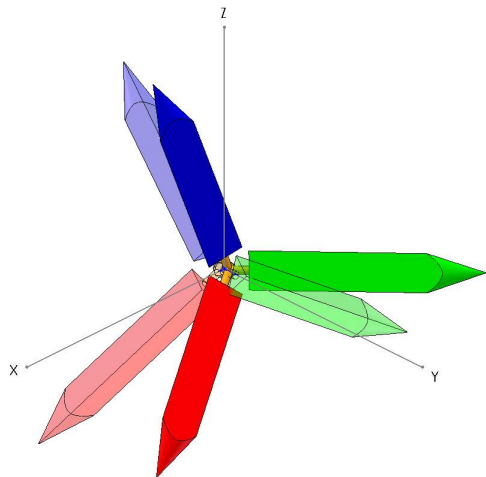
RPY Y 20 °



Euler X 10 °



RPY Z 30 °



## Installation

Developed and tested on 50g and 49g+. Rom revision 2.09.

If any previous version is installed on the calc, delete it (Filer, PURGE) before installation of v5.

Copy one of the files RC5basic.lib or RC5plus.lib to any of ports 0:IRAM, 1:ERAM or 2:FLASH on the calculator (Port 2 gives fastest execution). Two different methods of transfer from PC to calc:

1. PC to SD card, then COPY or MOVE from card to port.
2. From PC to HOME directory on calc, by USB Connectivity Kit. Then MOVE from HOME to port.

Finally make a warm start (Press ON and F3 at the same time, then release). Commands can then be accessed from the LIB menu, rightshift 2. For an alternative means of access the program RotC can be copied to a directory. And possibly attached to a key. See the Users Guide.

Note that lowercase letters in a PC file name will end up as uppercase letters on the calc, if transfer is via the SD card. Bit of a nuisance, but not a functional problem in this case.

## Versions

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**v2** November 07. Original submission to hpcalc.org

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**v3** March 08.

Quaternion input with negative scalar part not converted properly. Corrected.

Q<-->ΘV: Could give wrong result if angle input not within -180 .. 180 or -pi .. pi ranges.  
Corrected.

Method for checking if matrix is orthonormal was not OK. Changed.

Added RCLIM.

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**v4** March 09.

Argument checking reworked to reduce size. Other minor fixes and changes.

RCLIM: Changed default limit values and added explicit orthogonality limit. Added choice of help or error.

Basic and plus versions.

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**v5** March 10.

Simplified algorithms for factoring to angles. Improved precision at small rotation angles.

Changes for more consistent handling of unnormalized input.

Corrected sign error in parameter angle when mid Euler angle -90 deg and other angles cancels.

RCLIM: Added possibility to select if quaternion output should be allowed to be of any magnitude and to have negative scalar part.

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