

2² Factorial Design

José Maurício Schneedorf Ferreira da Silva

2026-02-13

Factorial design is an experimental statistical design used to assess to verify the significance of factors (e.g., pH, content, temperature, type) and levels in an experiment (e.g., value, type), without the need for complete trials with all samples. The most common designs involve two, three, or four levels. The *22fat1matrix* program was developed specifically to test two factors with 2 levels each.

1 Equation:

The statistical model for a 2² factorial experiment is:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

Where:

- $i = 1, 2$ (levels of factor A)
- $j = 1, 2$ (levels of factor B)
- $k = 1, 2, \dots, r$ (replicas)
- μ is the overall mean
- α_i is the effect of factor A
- β_j is the effect of factor B
- $(\alpha\beta)_{ij}$ is the interaction effect
- $\varepsilon_{ijk} \sim N(0, \sigma^2)$ is the experimental error

The above equation is also mirrored in the *multiple linear regression* model, as follows:

$$M(x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + error_{(x_1, x_2)}$$

Thus, a 2^2 factorial experiment can be solved algebraically using the multilinear adjustment model. In this case, solving by matrix relation:

Solving by a matrix relationship:

$$X^T \cdot y = \beta$$

Where:

- X^T = transpose of the planning coefficient matrix;
- y = response matrix of the experimental design;
- β = matrix of resulting coefficients.

The mathematical operation involved in factorial design resembles a multiple linear regression between variables, although using a *planning coefficient matrix* translated directly from the experimental values (Neto, 1996).

The resulting β matrix comprises:

```
"beta matrix:"
```

```
M - overall mean;  
A - effect of factor A;  
B - effect of factor B;  
AB - interaction effect.
```

2 Files:

1. Program
2. Example
3. Planning matrix
4. Source code
5. This tutorial

3 Usage and example

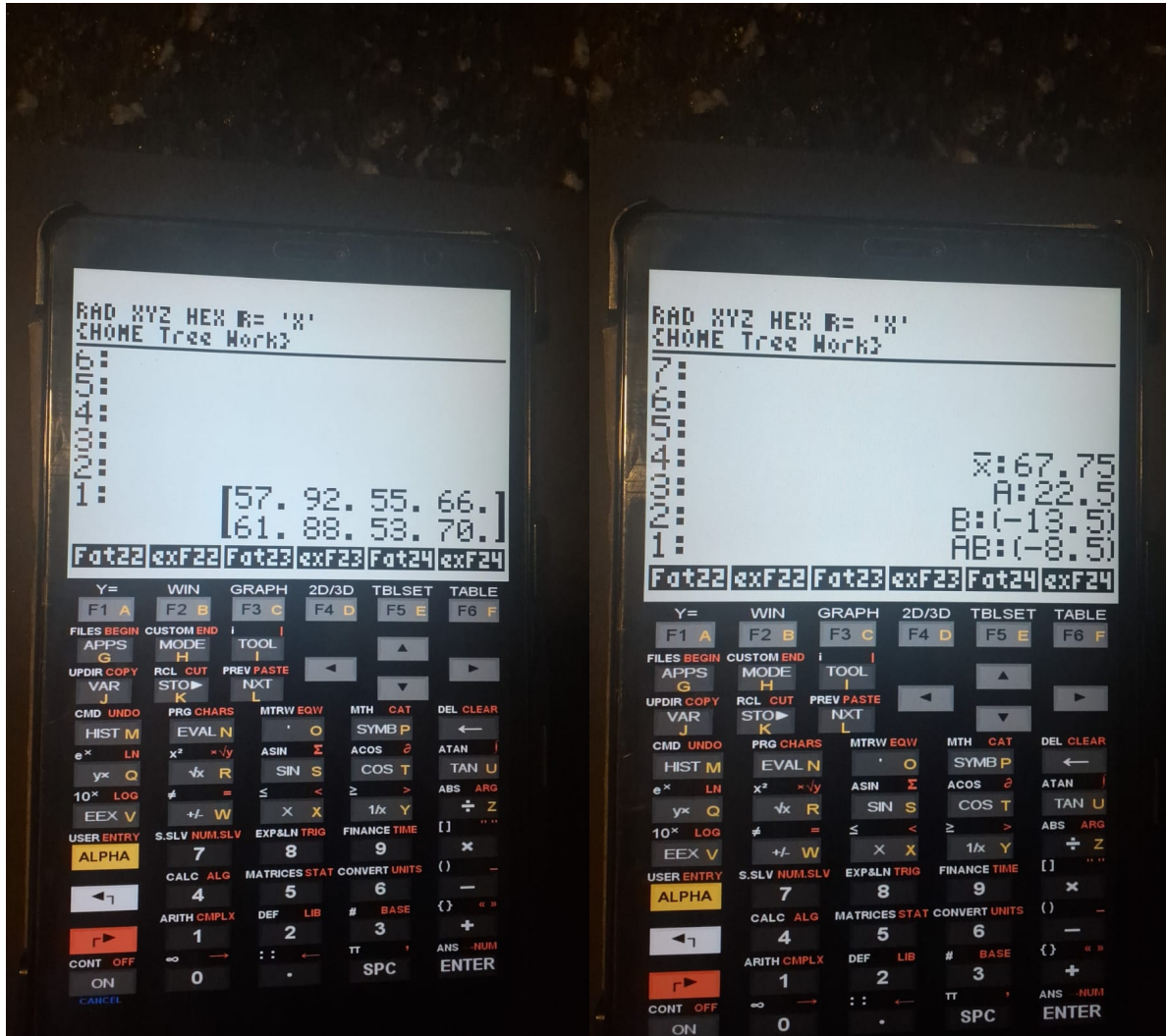
```
# Input:  
1. Enter the matrix containing the dataset;  
2. Run the program "22fat1matrix".  
  
# Output:  
1. Values of M, A, B, and AB.
```

Note: The *ex22fat* file included in *22fat1matrix.zip* refers only to the planning matrix of the example provided. For user-defined data, a new planning matrix must be constructed and stored in the variable *ex22fat* (Neto et al., 1996).

In the example (Neto *et al.*, 1995), factor *A* corresponds to the reaction temperature, whereas factor *B* represents the type of catalyst used in a chemical reaction. The values in the data matrix (input) correspond to reaction yield. In summary, the results indicate that increasing temperature led to a proportional increase in yield ($A = 22.5$), particularly when using catalyst *A* (while *B* was reduced), whereas changing the catalyst negatively affected yield ($B = -13.5$). The results are identical to those reported in the reference source (Neto *et al.*, 1996).

4 References:

1. Neto, B. B., Scarmino, I. S., Bruns, R. E. *Design and Optimization of Experiments*. 2nd ed., Unicamp Press, 1996.



(a) Insertion of data matrix.

(b) Results obtained from the 2^2 factorial design.

Figure 1: *22fat1matrix* program running on the Android version of the HP50G calculator (Go49gp), showing data matrix input and the results for the overall mean, main effects A and B, and interaction AB.

Author

José Maurício Schneedorf Ferreira da Silva
Dept. Biochemistry, Institute of Biomedical Sciences
Federal University of Alfenas - UNIFAL-MG, Brazil
Email: jose.dasilva@unifal-mg.edu.br

Site: <https://bioquanti.netlify.app>
Lattes: <http://lattes.cnpq.br/0436922594542722>
ORCID: 0000-0002-2031-6315