

2×2 Factorial Design

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2026-02-10

Factorial design is an experimental statistical design used to assess the significance of factors in an experiment (e.g., pH, concentration, temperature, type), without requiring fully exhaustive testing of all possible samples. The most common designs involve 2, 3, or 4 factors. The *fat1matrix* program was developed specifically to test two factors.

1 Equation:

$$M_{(x_1, x_2)} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \text{error}_{(x_1, x_2)}$$

Solving by a matrix relationship:

$$X^T \cdot y = \beta$$

Where:

- X^T = transpose of the contrast coefficient matrix;
- y = response matrix of the experimental design;
- β = matrix of resulting coefficients.

The mathematical operation involved in factorial design resembles a multiple linear regression between variables, although using a *contrast coefficient matrix* translated directly from the experimental values (Neto, 1996).

The resulting β matrix comprises:

```
"beta matrix:"  
  
M - overall mean;  
A - effect of factor A;  
B - effect of factor B;  
AB - interaction effect.
```

2 Files:

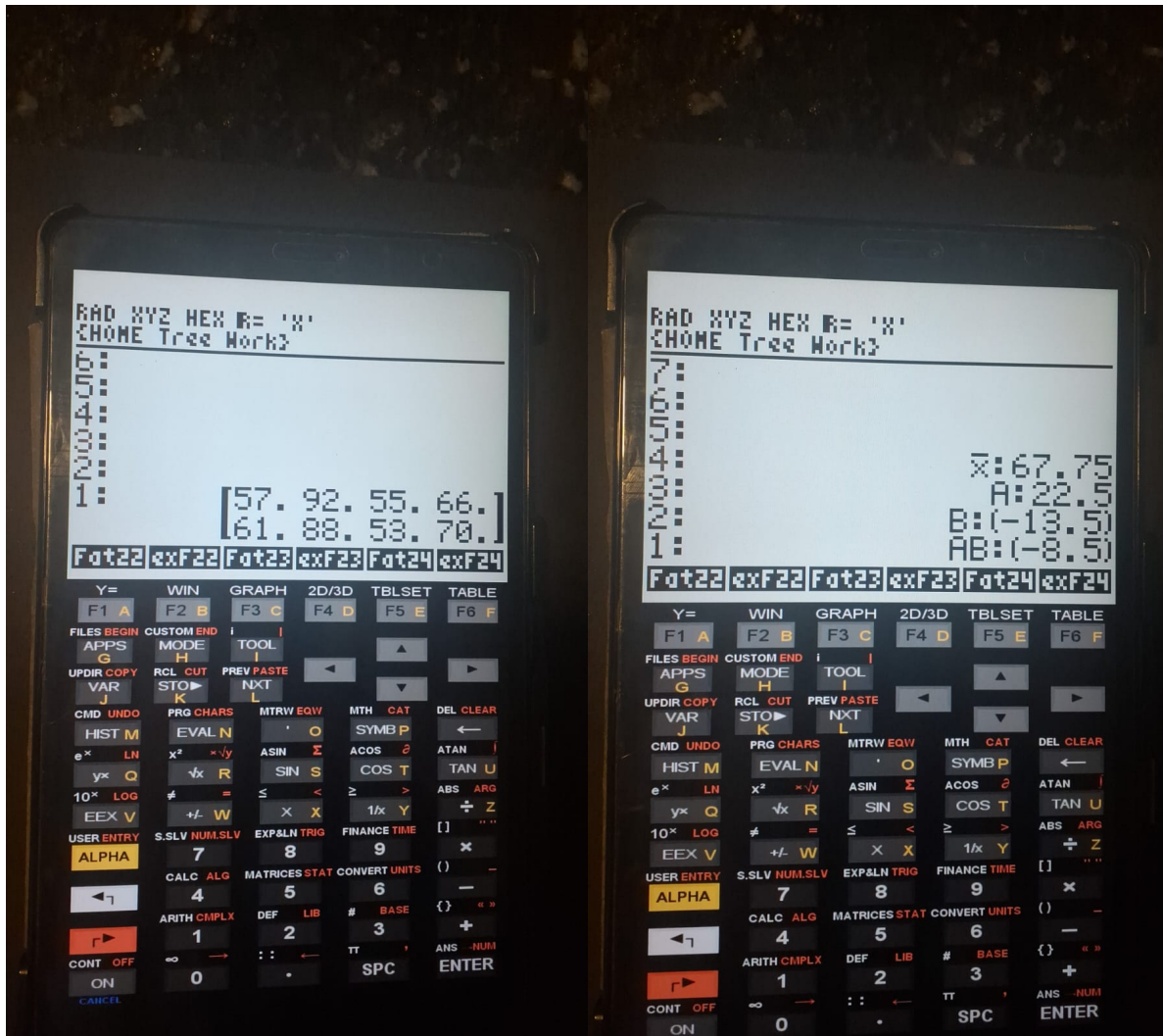
1. Program
2. Example
3. Contrast matrix for the example - c22 (HP50)
4. Contrast matrix (PDF)
5. Source code
6. This tutorial

3 Usage and example

```
# Input:  
1. Enter the matrix containing the dataset;  
2. Run the program "fat1matrix".  
  
# Output:  
1. Values of M, A, B, and AB.
```

Note: The *c22* file included in *fat1matrix.zip* refers only to the contrast matrix of the example provided. For user-defined data, a new contrast matrix must be constructed and stored in the variable *c22* (Neto et al., 1996).

In the example (Neto *et al.*, 1995), factor *A* corresponds to the reaction temperature, whereas factor *B* represents the type of catalyst used in a chemical reaction. The values in the data matrix (input) correspond to reaction yield. In summary, the results indicate that increasing



(a) Insertion of data matrix.

(b) Results obtained from the 2×2 factorial design.

Figure 1: *fat1matrix* program running on the Android version of the HP50G calculator (Go49gp), showing data matrix input and the results for the overall mean, main effects A and B, and interaction AB.

temperature led to a proportional increase in yield ($A = 22.5$), particularly when using catalyst A (while B was reduced), whereas changing the catalyst negatively affected yield ($B = -13.5$). The results are identical to those reported in the reference source (Neto *et al.*, 1996).

4 References:

1. Neto, B. B., Scarmino, I. S., Bruns, R. E. *Design and Optimization of Experiments*. 2nd ed., Unicamp Press, 1996.

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