RegPlus

HP 49g Graphing Calculator

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Modello di Regressione Normale

- $y_i = \beta_1 + \beta_2 x_{i1} + \beta_3 x_{i2} + \cdots + \beta_k x_{ik-1} + \varepsilon_i \quad \forall\, i = 1, 2, \ldots, n$
- $\varepsilon \sim N(0, \sigma^2 I_n)$
- $y \sim N(X\beta, \sigma^2 I_n)$
- σ^2 rappresenta la varianza d'errore

OLS Coefficients – Estimate

- Stima OLS
- $\hat{\beta} = (X^T X)^{-1} X^T y$

OLS Coefficients – Std. Error

- Standard Error delle stime OLS
- $s_{\hat{\beta}} = s\sqrt{\mathrm{diag}((X^T X)^{-1})}$

OLS Coefficients – t value

- Valori critici
- $t_{\hat{\beta}} = \hat{\beta} \,/\, s_{\hat{\beta}}$

OLS Coefficients – Pr(> |t|)

- *p*-value delle stime OLS
- $\Pr(>|t|) \; = \; 2 \;\; P(t_{n-k} \leq -|t_{\hat{\beta}}|)$

OLS Coefficients – lower 95%

- Estremo inferiore stime OLS al 95%
- $\hat{\beta}_i - z_{0.975}\, s_{\hat{\beta}_i} \quad \forall\, i = 1, 2, \ldots, k$

OLS Coefficients – upper 95%

- Estremo superiore stime OLS al 95%
- $\hat{\beta}_i + z_{0.975}\, s_{\hat{\beta}_i} \quad \forall\, i = 1, 2, \ldots, k$

OLS Coefficients – lower 99%

- Estremo inferiore stime OLS al 99%
- $\hat{\beta}_i - z_{0.995}\, s_{\hat{\beta}_i} \quad \forall\, i = 1, 2, \ldots, k$

OLS Coefficients – upper 99%

- Estremo superiore stime OLS al 99%
- $\hat{\beta}_i + z_{0.995}\, s_{\hat{\beta}_i} \quad \forall\, i = 1, 2, \ldots, k$

OLS Properties – std.dev

- Radice della stima OLS della varianza di errore s^2
- $s^2 = RSS\,/\,(n-k) = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2\,/\,(n-k)$

OLS Properties – cov.scaled

- Matrice di covarianza della OLS stima scalata per σ^2
- $(X^T X)^{-1}$

OLS Properties – cov.unscaled

- Matrice di covarianza della stima OLS
- $s^2 \left(X^T X\right)^{-1}$

OLS Properties – correlation

- Correlazione tra le stime OLS
- $r_{\hat{\beta}_i \hat{\beta}_j} = \dfrac{s^2 (X^T X)^{-1}_{(i,j)}}{s_{\hat{\beta}_i} s_{\hat{\beta}_j}} \quad \forall\, i, j = 1, 2, \ldots, k$

OLS Properties – vif

- Variance Inflation Factors
- $\left(1 - R^2_{x_j}\right)^{-1} \quad \forall j = 1, 2, \dots, k-1$
  dove $R^2_{x_j}$ rappresenta il valore di R^2 per il modello che presenta il regressore j-esimo come variabile dipendente

Anova Table – REG df

- Gradi di libertà della devianza spiegata
- $k - 1$

Anova Table – REG RSS

- Devianza spiegata
- $TSS - RSS = \sum_{i=1}^{n}(y_i - \bar{y})^2 - \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

Anova Table – REG MS

- Devianza spiegata media
- $(TSS - RSS) / (k-1) = \left(\sum_{i=1}^{n}(y_i - \bar{y})^2 - \sum_{i=1}^{n}(y_i - \hat{y}_i)^2\right) / (k-1)$

Anova Table – ERR df

- Gradi di libertà della devianza di errore
- $n - k$

Anova Table – ERR RSS

- Devianza di errore
- $RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$

Anova Table – ERR MS

- Devianza di errore media
- $RSS\,/\,(n-k) = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2\,/\,(n-k)$

Anova Table – TOT df

- Gradi di libertà della devianza totale
- $n - 1$

Anova Table – TOT RSS

- Devianza totale
- $TSS = \sum_{i=1}^{n} (y_i - \bar{y})^2$

Anova Table – TOT MS

- Devianza totale media
- $TSS\,/\,(n-1) = \sum_{i=1}^{n}(y_i - \bar{y})^2\,/\,(n-1)$

Anova Table – F

- Valore critico F
- $F = \frac{REG\,RSS\,/\,(k-1)}{ERR\,RSS\,/\,(n-k)}$

Anova Table – Pr(>F)

- *p*-value
- $\text{Pr(>F)} = \text{P}(F_{k-1,n-k} \geq F)$

Goodness of fit – r.squared

- Quota di devianza spiegata
- $R^2 = 1 - \frac{RSS}{TSS} = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$

Goodness of fit – adj.r.squared

- Correzione di R^2 per i gradi di libertà
- $R^2_{adj} = 1 - \frac{RSS\,(n-1)}{TSS\,(n-k)} = 1 - \frac{(n-1)\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{(n-k)\sum_{i=1}^{n}(y_i - \bar{y})^2}$

Goodness of fit – dwtest

- Test di *Durbin-Watson*
- $\sum_{i=2}^{n} (e_i - e_{i-1})^2 \,/\, RSS$

Fitted Values – yhat

- Valori fittati
- $\hat{y} = X(X^T X)^{-1} X^T y = H y$

Log-Likelihood – AIC

- Akaike Information Criterion
- $-2\hat{\ell} + 2(k+1)$

Log-Likelihood – BIC

- Bayesian Information Criterion
- $-2\,\hat{\ell} + (k+1)\log(n)$

Log-Likelihood – logLik

- Logverosimiglianza normale
- $\hat{\ell} = -n\,[\log(2\,\pi) + \log\,(RSS\,/\,n) + 1]\,/\,2$

Log-Likelihood – extractAIC

- Generalized Akaike Information Criterion
- $n \log(RSS\,/\,n) + 2\,k$

Drop single terms – RSS

- Devianza residua
- $RSS, RSS_{-x_j} \quad \forall j = 1, 2, \ldots, k-1$
  dove $\quad RSS_{-x_j}$ rappresenta la devianza residua del modello eliminata la variabile esplicativa x_j.

Drop single terms – Sum of Sq

- Differenza tra Devianze residue
- $RSS_{-x_j} - RSS \quad \forall j = 1, 2, \ldots, k-1$

Drop single terms – Df

- Gradi di libertà
- $\underbrace{1, 1, \ldots, 1}_{k-1 \text{ volte}}$

Drop single terms – AIC

- Akaike Information Criterion
- $n \log(RSS / n) + 2k$, $n \log\left(RSS_{-x_j} / n\right) + 2(k-1)$ $\forall j = 1, 2, \ldots, k-1$

Drop single terms – Cp

- Mallows Cp
- $k, \, (n-k)\, RSS_{-x_j} \,/\, RSS + 2\,(k-1) - n$
  $\forall j = 1, 2, \ldots, k-1$

Drop single terms – r.squared

- Quota di devianza spiegata
- $1 - \frac{RSS_{-x_j}}{TSS} \quad \forall j = 1, 2, \ldots, k-1$

Drop single terms – adj.r.squared

- Correzione di R^2 per i gradi di libertà
- $1 - \frac{RSS_{-x_j} \,/\, (n-k+1)}{TSS \,/\, (n-1)} \quad \forall j = 1, 2, \ldots, k-1$

Drop single terms – F value

- Valore critico F
- $F_{-x_j} = \frac{RSS_{-x_j} - RSS}{RSS \,/\, (n-k)} \quad \forall j = 1, 2, \ldots, k-1$

Drop single terms – Pr(>F)

- Valore critico F
- $\text{Pr(>F)} \;=\; \text{P}(F_{1,n-k} \geq F_{-x_j}) \quad \forall j = 1, 2, \ldots, k-1$

Residuals – resid

- Residui classici
- $e_i = y_i - \hat{y}_i \quad \forall\, i = 1, 2, \ldots, n$

Residuals – rstandard

- Residui standardizzati
- $rstandard_i = \frac{e_i}{s\sqrt{1-h_i}} \quad \forall\, i = 1, 2, \ldots, n$

Residuals – rstudent

- Residui studentizzati
- $rstudent_i = \frac{e_i}{s_{-i}\sqrt{1-h_i}} \quad \forall\, i = 1, 2, \ldots, n$

Breusch Pagan – BP

- Valore empirico
- $c = n \, \frac{v^T \, H \, v}{v^T \, v} \quad \text{dove} \; v_i = e_i^2 - RSS \, / \, n \quad \forall \, i = 1, 2, \ldots, n$

Breusch Pagan – df

- Gradi di libertà
- $k-1$

Breusch Pagan – p-value

- p-value
- $\mathrm{P}(\chi^2_{k-1} \geq c)$

Jarque Bera – JB

- Valore empirico
- $c = \frac{n}{6}\left(\frac{m_3}{m_2^{3/2}}\right)^2 + \frac{n}{24}\left(\frac{m_4}{m_2^2} - 3\right)^2$
  dove $m_k = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^k \quad \forall\, k = 2, 3, 4$

Jarque Bera – df

- Gradi di libertà
- 2

Jarque Bera – p-value

- p-value
- $\mathrm{P}(\chi^2_2 \geq c)$

Infl Measures – cookd

- Distanza di *Cook*
- $cd_i = \frac{h_i\, rstandard_i^2}{k\,(1-h_i)} = \frac{e_i^2}{k\, s^2}\, \frac{h_i}{(1-h_i)^2} \quad \forall\, i = 1, 2, \ldots, n$

Infl Measures – Pr(>F)

- Verifica significatività distanza di *Cook*
- $\Pr(>\mathrm{F}) \; = \; \mathrm{P}(F_{k-1,n-k} \geq cd_i) \quad \forall\, i = 1, 2, \ldots, n$

Infl Measures – mahalanobis

- Distanza di *Mahalanobis*
- $MD^2 = (x - \bar{x})^T S^{-1} (x - \bar{x})$

Infl Measures – hatvalues

- Valori di leva
- $h = \mathrm{diag}(H) = \mathrm{diag}\left(X\,(X^T X)^{-1} X^T\right)$

Infl Measures – sigma

- Stima di σ^2 tolta la i-esima osservazione
- $s^2_{-i} = s^2 \left(1 + \frac{1 - rstandard_i^2}{n-k-1}\right) \quad \forall\, i = 1, 2, \ldots, n$

Infl cutoff – dfb

- dfbetas
- $|dfbetas| > 1$

Infl cutoff – dffit

- Dffits
- $|\,Dffits\,| > 3\sqrt{k\,/\,(n-k)}$

Infl cutoff – cov.r

- Covratio
- $|1 - Covratio| > 3\,k\,/\,(n-k)$

Infl cutoff – hat

- Valori di leva
- $hat > 3\,k\,/\,n$

Outlier test – $max|rstudent|$

- Massimo residuo studentizzato assoluto
- $t = \max_i(|rstudent_i|)$

Outlier test – df

- Gradi di libertà
- $n - k - 1$

Outlier test – Pr(>t)

- p-value
- $\text{Pr}(>\text{t}) \; = \; 2\,P(\,t_{n-k-1} \leq -|\,t\,|)$

Outlier test – Observation

- Osservazione influente
- i tale che $t = rstudent_i$

Diagnostic – kappa

- Condition number
- $\frac{\max(\text{diag}(D))}{\min(\text{diag}(D))}$

  dove $\quad (X^T X) = U D V^T \quad \text{e} \quad U^T U = I_k = V^T V = V V^T$

Diagnostic – covratio

- Covratio
- $cr_i = (1 - h_i)^{-1} \left(1 + \frac{rstudent_i^2 - 1}{n - k}\right)^{-k} \quad \forall i = 1, 2, \ldots, n$

Diagnostic – dffits

- Dffits
- $rstudent_i \sqrt{\frac{h_i}{1-h_i}} \quad \forall i = 1, 2, \ldots, n$

Diagnostic – dfbeta

- Dfbeta
- $\hat{\beta}_j - \hat{\beta}_{j\,(-i)} = e_i\,(1 - h_i)^{-1}\,(X^T X)_j^{-1}\,X_i^T$
  $\forall i = 1, 2, \ldots, n \quad \forall j = 1, 2, \ldots, k$

Diagnostic – dfbetas

- Dfbetas
- $$\frac{\hat{\beta}_j - \hat{\beta}_{j\,(-i)}}{s_{\hat{\beta}_j - \hat{\beta}_{j\,(-i)}}} = \frac{e_i\,(1-h_i)^{-1}\,(X^T X)_j^{-1}\,X_i^T}{s_{-i}\sqrt{(X^T X)_{j,j}^{-1}}}$$
  $$\forall i = 1, 2, \ldots, n \quad \forall j = 1, 2, \ldots, k$$