

Pipe flow with friction losses – solutions using HP and TI calculators

By Gilberto E. Urroz, October 2005

1. Darcy-Weisbach Equation and friction factor

The basic equation governing friction losses in a pipeline is the Darcy-Weisbach equation:

$$h_f = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g}, \text{ or } h_f = \frac{8 \cdot f \cdot L \cdot Q^2}{\pi^2 \cdot g \cdot D^5} \quad (1)$$

where f is the friction factor, e is the absolute roughness (or equivalent sand roughness) of the pipe, D is the pipe diameter, R is the Reynolds number, L is the pipe length, V is the flow velocity, Q is the discharge, and g is the acceleration of gravity. The discharge Q and the flow velocity V are related by the continuity equation, namely,

$$V = \frac{4 \cdot Q}{\pi \cdot D^2}, \text{ or } Q = V \cdot \frac{\pi \cdot D^2}{4}. \quad (2).$$

The Reynolds number is defined as

$$R = \frac{\rho V D}{\mu} = \frac{V D}{\nu}, \text{ or } R = \frac{4 \cdot \rho \cdot Q}{\pi \cdot \mu \cdot D} = \frac{4 \cdot Q}{\pi \cdot \nu \cdot D}. \quad (3)$$

where ρ is the fluid density, μ is its dynamic (or absolute) viscosity, and $\nu = \mu/\rho$ is its kinematic viscosity [units = m^2/s or ft^2/s].

The friction factor f is a function of the relative roughness e/D and of the Reynolds number R . Values of f can be obtained from the Moody diagram that shows curves of constant relative roughness for a range of values of the Reynolds number and the corresponding friction factors. The diagram includes also the laminar friction factor given by Stokes' equation:

$$f = \frac{64}{R}. \quad (4)$$

Function DARC Y in HP calculators

The HP 48 G and HP 49 G series calculators provide function $DARC Y(ee/D,R)^1$ to calculate the friction factor f for turbulent flows. In the HP 48 G, function $DARC Y$ is available by using the keystrokes:

[←][EQ LIB][UTILS]

The resulting menu will show the functions:

¹ Here we use ee instead of e because the HP calculators would interpret e as $\exp(1)$.

MINE:	Minesweeper game
ZFACT:	Gas compressibility Z factor function
FANNI:	Fanning's friction factor (4 times D-W's friction factor)
DARCY:	The function of interest (Darcy-Weisbach's friction factor)
F0λ:	Black-body emissive power function
SIDEN:	Silicon intrinsic density

In the HP 49 G, HP 49 G+, and HP 48GII, function *DARCY* is available through the function catalog, [CAT], or you could simply type the name of the function.

To calculate the friction factor with function *DARCY* using the HP 48 G series calculators, or the HP 49 G, HP 49 G+, and HP 48GII calculators in *RPN* (Reverse Polish Notation), enter the values of e/D and \mathbf{R} in the stack, and then invoke function *DARCY*. If using the HP 49 G, HP 49 G+, and HP 48GII calculators in *ALG* (algebraic) mode, enter the expression *DARCY*(value of e/D , value of \mathbf{R}).

As an example, calculate the friction factors for the following combinations of friction factors and Reynolds numbers. Verify the values listed in Table 1.

Table 1. Friction factors calculated with HP's function *DARCY*

e/D	\mathbf{R}	f
0.001	1.23×10^5	0.0217
0.0001	3.32×10^6	0.0125
0.00001	1.03×10^8	0.0081
0.0005	5.23×10^7	0.0167
0.0032	8.25×10^4	0.0281

Function *DARCY* implements the *Coolebrook-White* equation, as well as the laminar equation, to calculate the friction factor. Using function *DARCY* in the HP calculators is equivalent to reading f out of the Moody diagram knowing the values of e/D and \mathbf{R} .

Equations for the friction factor

The Coolebrook-White equation, shown below, is an implicit equation in f , and, therefore, not amenable to direct solution.

$$\frac{1}{\sqrt{f}} = -2 \cdot \log \left(\frac{e}{3.7 \cdot D} + \frac{2.51}{\mathbf{R} \cdot \sqrt{f}} \right) \quad (5)$$

Modern alternatives for the Coolebrook-White equation that are explicit in f include Haaland's equation and Swamee-Jain's equation. Haaland's equation is:

$$\frac{1}{\sqrt{f}} = -1.8 \cdot \log \left[\left(\frac{e}{3.7 \cdot D} \right)^{1.11} + \frac{6.9}{\mathbf{R}} \right], \quad (6)$$

or, equivalently,

$$f = \frac{0.3086}{\log^2 \left[\left(\frac{e}{3.7 \cdot D} \right)^{1.11} + \frac{6.9}{\mathbf{R}} \right]}. \quad (7)$$

Swamee-Jain equation is given by:

$$f = \frac{0.25}{\log^2 \left(\frac{e}{3.7D} + \frac{5.74}{\mathbf{R}^{0.9}} \right)}. \quad (8)$$

In equations (5) through (8), \log stands for the logarithm of base 10, and $\log^2() \equiv [\log()]^2$.

Implementing Haaland's and Swamee-Jain equations in HP calculators

As alternatives for the function *DARCY* in the HP calculators, one can define functions *fHA* and *fSJ* to implement the explicit form of the Haaland's and Swamee-Jain's equations, respectively. In order to keep all the pipe-related functions and equations together, I suggest creating a sub-directory, call it *PIPES*, within the *HOME* directory of your calculator. In order to define the functions you need to use the key *DEF* with the following arguments:

```
' fHA (eD , R) = 0 . 3086 / ( LOG ( ( eD / 3 . 7 ) ^ 1 . 11 + 6 . 9 / R ) ) ^ 2 '
' fSJ (eD , R) = 0 . 25 / ( LOG ( eD / 3 . 7 + 5 . 74 / R ^ 0 . 9 ) ) ^ 2 '
```

In these definitions *eD* stands for the relative roughness (*e/D*) and *R* stands for the Reynolds number **R**. After defining these functions there will be soft-menu keys labeled [fHA] and [fSJ] in your calculator. To see the variables available in your *PIPES* sub-directory you may have to press the [VAR] key.

The operation of these two user-defined functions, namely, *fHA* and *fSJ*, is very similar to the operation of function *DARCY*. To verify the implementation of these functions in HP calculators check the following values returned by the functions for the parameters *e/D* and **R** as given in Table 2.

Table 2. Values of *f* calculated with user-defined functions *fHA* and *fSJ*

<i>e/D</i>	R	<i>fHA</i>	<i>fSJ</i>
0.001	1.23×10^5	0.0216	0.0219
0.0001	3.32×10^6	0.0125	0.0126
0.00001	1.03×10^8	0.0082	0.0082
0.0005	5.23×10^7	0.0167	0.0167
0.0032	8.25×10^4	0.0280	0.0284

Implementing Haaland's and Swamee-Jain equations in TI calculators

In the TI-89 or TI-92 calculators, you can program functions *fha(k,r)* and *fsj(k,r)* to calculate the friction factor using Haaland's and Swamee-Jain's equations, respectively.

In these definitions, k represents the relative roughness (e/D) and r represents the Reynolds number. To program function fha , select the *Program Editor* in your calculator, and then select the option 3:New to enter a new function. Select:

Type: 2:Function
 Folder: (your favorite folder, e.g., one called “fluids”)
 Variable: fha

and press [Enter]. Edit the function such that the editor’s window looks as follows:

```
:fha(k,r)
:Func
:.3086/(log((k/3.7)^1.11+6.9/r)^2)
:EndFunc
```

Similarly, to implement function fsj select the *Program Editor* in your calculator, and then select the option 3:New to enter a new function. Select:

Type: 2:Function
 Folder: (your favorite folder, e.g., one called “fluids”)
 Variable: fsj

and press [Enter]. Edit the function such that the editor’s window looks as follows:

```
:fsj(k,r)
:Func
: .25/(log(k/(3.75)+5.74/r^(.9)))^2
:EndFunc
```

Press [HOME] to return to the HOME screen. At this point you are ready to use functions fha and fsj to calculate friction factors. To load the function name in the HOME screen entry line, you can either type the function name fha or fsj , or use [2ND][VAR-LINK] and select the function from the list thus produced. The function name must be followed by a set of parentheses including the values of $k = e/D$ and Re separated by commas. Press [ENTER] to evaluate the function. After implementing functions fha and fsj in your TI calculator, verify the results shown in Table 2.

2. Types of problems involving the Darcy-Weisbach equation for friction losses

The textbook by Finnemore and Franzini identifies three types of problems using the Darcy-Weisbach equation, namely:

1. Head loss problem: calculate h_f given D , Q or V , and g , L , e , ν .
2. Discharge problem: calculate Q or V , given D , h_f and g , L , e , ν .
3. Sizing problem: calculate D , given Q , h_f and g , L , e , ν .

Examples of the three types of problems are shown next:

Problem [1]. Given $D = 0.3 \text{ ft}$, $Q = 0.20 \text{ cfs}$, $g = 32.2 \text{ ft/s}^2$, $L = 1000 \text{ ft}$, $e = 0.002 \text{ in} = 0.000166 \text{ ft}$, and $\nu = 1.13 \times 10^{-5} \text{ ft}^2/\text{s}$, find h_f .

Problem [2]. Given $D = 0.7 \text{ ft}$, $h_f = 15 \text{ ft}$, $g = 32.2 \text{ ft/s}^2$, $L = 750 \text{ ft}$, $e = 0.005 \text{ in} = 0.000416 \text{ ft}$, and $\nu = 1.2 \times 10^{-5} \text{ ft}^2/\text{s}$, find Q .

Problem [3]. Given $Q = 3 \text{ cfs}$, $h_f = 10 \text{ ft}$, $g = 32.2 \text{ ft/s}^2$, $L = 1500 \text{ ft}$, $e = 0.01 \text{ in} = 0.000833 \text{ ft}$, and $\nu = 1.5 \times 10^{-5} \text{ ft}^2/\text{s}$, find D .

These problems are solved next using a variety of approaches.

Solution of sample problems using the Moody diagram

Solution to Problem [1]: For $e/D \approx 0.0006$, $\mathbf{R} = 4Q/(\pi\nu D) = 7.5 \times 10^4$, the Moody diagram shows $f \approx 0.022$, therefore, $\boxed{h_f = 8fLQ^2/(\pi^2 g D^5) = 9.12 \text{ ft}}$.

Solution to Problem [2]: $e/D \approx 0.0006$. From $h_f = 8fLQ^2/(\pi^2 g D^5)$, we get $fQ^2 = \pi^2 g D^5 h_f / (8L) = 0.1335$, from which $Q = 0.365/\sqrt{f}$ (A). Also, $\mathbf{R} = 4Q/(\pi\nu D) = 1.52 \times 10^5 Q$ (B). An iterative solution proceeds as follows:

- Assume $f = 0.03$, (A) gives $Q = 0.365/\sqrt{0.03} = 2.107 \text{ cfs}$, $\mathbf{R} = 3.2 \times 10^5$
- Moody: $f = 0.019$, (A) gives $Q = 0.365/\sqrt{0.019} = 2.65 \text{ cfs}$, $\mathbf{R} = 4.03 \times 10^5$
- Moody: $f = 0.019$, convergence achieved, thus $\boxed{Q = 2.65 \text{ cfs}}$.

Solution to Problem [3]: From $h_f = 8fLQ^2/(\pi^2 g D^5)$, we get $D^5/f = 8LQ^2/(\pi^2 g h_f) = 33.98$, from which $D = 2.02 f^{1/5}$ (A). The relative roughness is $e/D = 0.000833/D$ (B), and the Reynolds number is $\mathbf{R} = 4Q/(\pi\nu D) = 2.55 \times 10^5/D$ (C). An iterative procedure is implemented as follows:

- Assume $f = 0.03$, (A) $D = 1.00 \text{ ft}$, (B) $e/D = 0.0008$, (C) $\mathbf{R} = 2.55 \times 10^5$
- Moody: $f = 0.020$, (A) $D = 0.923 \text{ ft}$, (B) $e/D = 0.0009$, (C) $\mathbf{R} = 2.76 \times 10^5$
- Moody: $f = 0.021$, (A) $D = 0.93 \text{ ft}$, (B) $e/D \approx 0.0009$, (C) $\mathbf{R} = 2.74 \times 10^5$
- Moody: $f = 0.021$, convergence achieved, thus $\boxed{D = 0.93 \text{ ft}}$.

Solution using functions DARC Y, fHA and fSJ instead of the Moody diagram

Use of the Moody diagram requires us to read the values of f from the diagram for known values of e/D and \mathbf{R} . Functions *DARC Y* (in HP calculators only), *fHA* (or *fha*) and *fSJ* (or *fsj*) can be used to calculate the values of f instead of reading them out of the Moody diagram.

Solution of sample problems using the DARC Y function

The *DARC Y* function being available only in the HP calculators, these solutions cannot be implemented in the TI calculator. Solutions using functions *fHA* and *fSJ* for both calculators will be presented in subsequent sections. The following solutions used function *DARC Y* to calculate the friction factor f :

Solution to Problem [1]: Given $D = 0.3 \text{ ft}$, $Q = 0.20 \text{ cfs}$, $g = 32.2 \text{ ft/s}^2$, $L = 1000 \text{ ft}$, $e = 0.002 \text{ in} = 0.000166 \text{ ft}$, and $\nu = 1.13 \times 10^{-5} \text{ ft}^2/\text{s}$, find h_f .

For $e/D = 0.00059$, $\mathbf{R} = 4Q/(\pi\nu D) = 7.5 \times 10^4$, the *DARC Y* function shows $f = 0.0194$, therefore, $\boxed{h_f = 8fLQ^2/(\pi^2 g D^5) = 8.04 \text{ ft}}$.

Solution to Problem [2]: Given $D = 0.7 \text{ ft}$, $h_f = 15 \text{ ft}$, $g = 32.2 \text{ ft/s}^2$, $L = 750 \text{ ft}$, $e = 0.005 \text{ in} = 0.000416 \text{ ft}$, and $\nu = 1.2 \times 10^{-5} \text{ ft}^2/\text{s}$, find Q .

$e/D = 0.00059$. From $h_f = 8fLQ^2/(\pi^2 g D^5)$, we get $fQ^2 = \pi^2 g D^5 h_f / (8L) = 0.1335$, from which $Q = 0.365/\sqrt{f}$ (A). Also, $\mathbf{R} = 4Q/(\pi \nu D) = 1.52 \times 10^5 Q$ (B). An iterative solution proceeds as follows:

- Assume $f = 0.03$, (A) gives $Q = 0.365/\sqrt{0.03} = 2.107 \text{ cfs}$, $\mathbf{R} = 3.2 \times 10^5$
- DARCY: $f = 0.0186$, (A) gives $Q = 0.365/\sqrt{0.0186} = 2.67 \text{ cfs}$, $\mathbf{R} = 4.05 \times 10^5$
- DARCY: $f = 0.0184$, (A) gives $Q = 0.365/\sqrt{0.0184} = 2.69 \text{ cfs}$, $\mathbf{R} = 4.09 \times 10^5$
- DARCY: $f = 0.0184$, convergence achieved, thus $\boxed{Q = 2.69 \text{ cfs}}$.

Solution to Problem [3]: Given $Q = 3 \text{ cfs}$, $h_f = 10 \text{ ft}$, $g = 32.2 \text{ ft/s}^2$, $L = 1500 \text{ ft}$, $e = 0.01 \text{ in} = 0.000833 \text{ ft}$, and $\nu = 1.5 \times 10^{-5} \text{ ft}^2/\text{s}$, find Q .

From $h_f = 8fLQ^2/(\pi^2 g D^5)$, we get $D^5/f = 8LQ^2/(\pi^2 g h_f) = 33.98$, from which $D = 2.02 \text{ ft}^{4/5}$ (A). The relative roughness is $e/D = 0.000833/D$ (B), and the Reynolds number is $\mathbf{R} = 4Q/(\pi \nu D) = 2.55 \times 10^5/D$ (C). An iterative procedure is implemented as follows:

- Assume $f = 0.03$, (A) $D = 1.00 \text{ ft}$, (B) $e/D = 0.000833$, (C) $\mathbf{R} = 2.55 \times 10^5$
- DARCY: $f = 0.0200$, (A) $D = 0.924 \text{ ft}$, (B) $e/D = 0.0009$, (C) $\mathbf{R} = 2.35 \times 10^5$
- DARCY: $f = 0.0204$, (A) $D = 0.927 \text{ ft}$, (B) $e/D \approx 0.0009$, (C) $\mathbf{R} = 2.36 \times 10^5$
- DARCY: $f = 0.0204$, convergence achieved, thus $\boxed{D = 0.927 \text{ ft} \approx 0.93 \text{ ft}}$.

Solution of sample problems using the fHA or fha function

Function *fHA* or *fha* implement Haaland's equation (7) to calculate the friction factor. The solutions to the three sample problems using this equation for f , instead of the Moody diagram, is shown next:

Solution to Problem [1]: Given $D = 0.3 \text{ ft}$, $Q = 0.20 \text{ cfs}$, $g = 32.2 \text{ ft/s}^2$, $L = 1000 \text{ ft}$, $e = 0.002 \text{ in} = 0.000166 \text{ ft}$, and $\nu = 1.13 \times 10^{-5} \text{ ft}^2/\text{s}$, find h_f .

For $e/D = 0.00059$, $\mathbf{R} = 4Q/(\pi \nu D) = 7.5 \times 10^4$, function *fHA* shows $f \approx 0.0212$, therefore, $\boxed{h_f = 8fLQ^2/(\pi^2 g D^5) = 8.73 \text{ ft}}$.

Solution to Problem [2]: Given $D = 0.7 \text{ ft}$, $h_f = 15 \text{ ft}$, $g = 32.2 \text{ ft/s}^2$, $L = 750 \text{ ft}$, $e = 0.005 \text{ in} = 0.000416 \text{ ft}$, and $\nu = 1.2 \times 10^{-5} \text{ ft}^2/\text{s}$, find Q .

$e/D = 0.00059$. From $h_f = 8fLQ^2/(\pi^2 g D^5)$, we get $fQ^2 = \pi^2 g D^5 h_f / (8L) = 0.1335$, from which $Q = 0.365/\sqrt{f}$ (A). Also, $\mathbf{R} = 4Q/(\pi \nu D) = 1.52 \times 10^5 Q$ (B). An iterative solution proceeds as follows:

- Assume $f = 0.03$, (A) gives $Q = 0.365/\sqrt{0.03} = 2.107 \text{ cfs}$, $\mathbf{R} = 3.2 \times 10^5$
- *fHA*: $f = 0.0196$, (A) gives $Q = 0.365/\sqrt{0.0186} = 2.67 \text{ cfs}$, $\mathbf{R} = 3.96 \times 10^5$
- *fHA*: $f = 0.0184$, (A) gives $Q = 0.365/\sqrt{0.0184} = 2.69 \text{ cfs}$, $\mathbf{R} = 4.02 \times 10^5$
- *fHA*: $f = 0.0183$, convergence achieved, thus $\boxed{Q = 2.69 \text{ cfs}}$.

Solution to Problem [3]: Given $Q = 3$ cfs, $h_f = 10$ ft, $g = 32.2$ ft/s², $L = 1500$ ft, $e = 0.01$ in $= 0.000833$ ft, and $\nu = 1.5 \times 10^{-5}$ ft²/s, find Q .

From $h_f = 8fLQ^2/(\pi^2 g D^5)$, we get $D^5/f = 8LQ^2/(\pi^2 g h_f) = 33.98$, from which $D = 2.02$ ft^{1/5} (A). The relative roughness is $e/D = 0.000833/D$ (B), and the Reynolds number is $\mathbf{R} = 4Q/(\pi \nu D) = 2.55 \times 10^5/D$ (C). An iterative procedure is implemented as follows:

- Assume $f = 0.03$, (A) $D = 1.00$ ft, (B) $e/D = 0.000833$, (C) $\mathbf{R} = 2.55 \times 10^5$
- f_{HA} : $f = 0.0198$, (A) $D = 0.88$ ft, (B) $e/D = 0.00095$, (C) $\mathbf{R} = 2.89 \times 10^5$
- f_{HA} : $f = 0.0203$, (A) $D = 0.93$ ft, (B) $e/D \approx 0.0009$, (C) $\mathbf{R} = 2.74 \times 10^5$
- f_{HA} : $f = 0.0202$, convergence achieved, thus $\boxed{D = 0.93 \text{ ft}}$.

Solution of sample problems using the fSJ or fsj function

Function *fSJ* or *fsj* implements Swamme-Jain's equation (8) to calculate the friction factor. The solutions to the three sample problems using this equation for f , instead of the Moody diagram, is shown next:

Solution to Problem [1]: Given $D = 0.3$ ft, $Q = 0.20$ cfs, $g = 32.2$ ft/s², $L = 1000$ ft, $e = 0.002$ in $= 0.000166$ ft, and $\nu = 1.13 \times 10^{-5}$ ft²/s, find h_f .

For $e/D = 0.00059$, $\mathbf{R} = 4Q/(\pi \nu D) = 7.5 \times 10^4$, the Moody diagram shows $f \approx 0.0216$, therefore, $\boxed{h_f = 8fLQ^2/(\pi^2 g D^5) = 8.89 \text{ ft}}$.

Solution to Problem [2]: Given $D = 0.7$ ft, $h_f = 15$ ft, $g = 32.2$ ft/s², $L = 750$ ft, $e = 0.005$ in $= 0.000416$ ft, and $\nu = 1.2 \times 10^{-5}$ ft²/s, find Q .

$e/D = 0.00059$. From $h_f = 8fLQ^2/(\pi^2 g D^5)$, we get $fQ^2 = \pi^2 g D^5 h_f / (8L) = 0.1335$, from which $Q = 0.365/\sqrt{f}$ (A). Also, $\mathbf{R} = 4Q/(\pi \nu D) = 1.52 \times 10^5 Q$ (B). An iterative solution proceeds as follows:

- Assume $f = 0.03$, (A) gives $Q = 0.365/\sqrt{0.03} = 2.107$ cfs, $\mathbf{R} = 3.2 \times 10^5$
- f_{SJ} : $f = 0.0188$, (A) gives $Q = 0.365/\sqrt{0.0186} = 2.66$ cfs, $\mathbf{R} = 4.04 \times 10^5$
- f_{SJ} : $f = 0.0186$, (A) gives $Q = 0.365/\sqrt{0.0184} = 2.68$ cfs, $\mathbf{R} = 4.07 \times 10^5$
- f_{SJ} : $f = 0.0186$, convergence achieved, thus $\boxed{Q = 2.68 \text{ cfs}}$.

Solution to Problem [3]: Given $Q = 3$ cfs, $h_f = 10$ ft, $g = 32.2$ ft/s², $L = 1500$ ft, $e = 0.01$ in $= 0.000833$ ft, and $\nu = 1.5 \times 10^{-5}$ ft²/s, find Q .

From $h_f = 8fLQ^2/(\pi^2 g D^5)$, we get $D^5/f = 8LQ^2/(\pi^2 g h_f) = 33.98$, from which $D = 2.02$ ft^{1/5} (A). The relative roughness is $e/D = 0.000833/D$ (B), and the Reynolds number is $\mathbf{R} = 4Q/(\pi \nu D) = 2.55 \times 10^5/D$ (C). An iterative procedure is implemented as follows:

- Assume $f = 0.03$, (A) $D = 1.00$ ft, (B) $e/D = 0.000833$, (C) $\mathbf{R} = 2.55 \times 10^5$
- f_{SJ} : $f = 0.0200$, (A) $D = 0.92$ ft, (B) $e/D = 0.0009$, (C) $\mathbf{R} = 2.77 \times 10^5$
- f_{SJ} : $f = 0.0204$, (A) $D = 0.93$ ft, (B) $e/D = 0.0009$, (C) $\mathbf{R} = 2.74 \times 10^5$
- f_{SJ} : $f = 0.0204$, convergence achieved, thus $\boxed{D = 0.93 \text{ ft}}$.

The following table summarizes the solutions of the three sample problems using (a) the Moody diagram, (b) the *DARCY* function in the HP calculators, (c) the *fHA* or *fha* function, and (d) the *fSJ* or *fsj* function.

Table 3. Solutions to sample problems through different methods of obtaining *f*

	Moody	DARCY	fHA	fSJ
Problem [1]	$f = 0.022$ $hf = 9.05 \text{ ft}$	$f = 0.0194$ $hf = 8.04 \text{ ft}$	$f = 0.0212$ $hf = 8.73 \text{ ft}$	$f = 0.0216$ $hf = 8.89 \text{ ft}$
Problem [2]	$f = 0.019$ $Q = 2.65 \text{ cfs}$	$f = 0.0184$ $Q = 2.69 \text{ cfs}$	$f = 0.0183$ $Q = 2.69 \text{ cfs}$	$f = 0.0186$ $Q = 2.68 \text{ cfs}$
Problem [3]	$f = 0.021$ $D = 0.93 \text{ ft}$	$f = 0.0204$ $D = 0.927 \text{ ft}$	$f = 0.0202$ $D = 0.93 \text{ ft}$	$f = 0.0204$ $D = 0.93 \text{ ft}$

3. Combining the Darcy-Weisbach equation with various equations for *f* **Darcy-Weisbach and Coolebrook-White equation**

We can combine the Darcy-Weisbach equation in terms of the velocity V (1) and the Coolebrook-White equation (5) as follows. First, from (1) we get

$$V = \sqrt{\frac{2gDh_f}{L}} \cdot \frac{1}{\sqrt{f}}, \quad (9)$$

and

$$\frac{1}{\sqrt{f}} = V \sqrt{\frac{L}{2gDh_f}}. \quad (10)$$

The last term within the logarithmic function in (5) is written as follows in terms of the definition of **R** (3) and (10):

$$\frac{2.51}{\mathbf{R}\sqrt{f}} = 2.51 \cdot \frac{1}{\mathbf{R}} \cdot \frac{1}{\sqrt{f}} = 2.51 \cdot \frac{v}{VD} \cdot V \sqrt{\frac{L}{2gDh_f}} = 2.51 \cdot \frac{v}{D} \sqrt{\frac{L}{2gDh_f}}. \quad (11)$$

Replacing $1/\sqrt{f}$ in (5) within equation (9), with the result of (11) included, results in the equation [see Eq. 8.56a, page 293, in Finnemore and Franzini]:

$$V = -2.828 \sqrt{\frac{gDh_f}{L}} \cdot \log \left(0.27 \cdot \frac{e}{D} + 1.775 \cdot \frac{v}{D} \cdot \sqrt{\frac{L}{gDh_f}} \right). \quad (12)$$

Equation (12) can be re-written in terms of the discharge by using the continuity equation (2) [see Eq. 8.56b, page 293, in Finnemore and Franzini]:

$$Q = -2.22 \sqrt{\frac{gD^5 h_f}{L}} \cdot \log \left(0.27 \cdot \frac{e}{D} + 1.775 \cdot \frac{\nu}{D} \cdot \sqrt{\frac{L}{gDh_f}} \right). \quad \text{DWCWQ(13)}$$

Equation (13) is explicit in Q , thus, it is appropriate for a direct solution of problems of type 2, *the discharge problem*. Solutions of type 1 (*head loss*) and type 3 (*sizing*) problems using equation (13) require the use of numerical solutions. Equation (13) is referred to by the name DWCWQ, i.e., *Darcy-Weisbach + Coolebrook-White* in terms of Q .

Darcy-Weisbach and Haaland equations

Combining the Darcy-Weisbach equation in terms of the discharge Q , equation (1), and the Haaland equation (6), and combining numeric terms results in:

$$Q = -2.0 \sqrt{\frac{gD^5 h_f}{L}} \cdot \log \left(0.234 \cdot \left(\frac{e}{D} \right)^{1.11} + 5.42 \cdot \frac{\nu D}{Q} \right). \quad \text{DWHAQ(14)}$$

This equation is implicit in Q and D , but explicit in h_f . Thus, this equation is ideal for solving type 1 problems. Type 2 and 3 problems, however, will require a numerical solution. Equation (14) is referred to by the name DWHAQ, i.e., *Darcy-Weisbach + HAaland* in terms of Q .

Darcy-Weisbach and Swamee-Jain equations

Combining the Darcy-Weisbach equation in terms of the discharge Q , equation (1), and the Swamee-Jain equation (8) requires taking the square root of the friction factor. In such operation we keep the negative value of the square root as shown in the following equation:

$$Q = -2.22 \sqrt{\frac{gD^5 h_f}{L}} \cdot \log \left(0.27 \cdot \frac{e}{D} + 4.62 \cdot \left(\frac{\nu D}{Q} \right)^{0.9} \right). \quad \text{DWSJQ(15)}$$

The reason for using the negative value in equation (15) is that the logarithmic function has an argument that is smaller than 1, thus producing negative logarithms. Since the discharge in (15) must be a positive quantity, the use of the negative sign in that equation is needed.

As with equation (14), equation (15) is implicit in Q and D , but explicit in h_f . Thus, this equation is ideal for solving type 1 problems. Type 2 and 3 problems, however, will require a numerical solution. Equation (15) is referred to by the name DWSJQ, i.e., *Darcy-Weisbach + Swamee-Jain* in terms of Q .

Solution of sample problems using equations (13) through (15)

The solution of sample problems [1] through [3] using equations (13) through (15) can be implemented using the numerical solvers in the HP and TI calculators. In order to

activate such solvers we need to store the equations into variables. For example, in the HP calculators, within sub-directory PIPES, we can define variables *DWCWQ*, *DWHAQ*, and *DWSJQ*, which store the following expressions:

DWCWQ:

$$Q = -2.22 \sqrt{(g \cdot D^5 \cdot hf / L) \cdot \text{LOG}(0.27 \cdot ee / D + 1.775 \cdot Nu / D \sqrt{L / (g \cdot D \cdot hf)})}$$

DWHAQ:

$$Q = -2.0 \sqrt{(g \cdot D^5 \cdot hf / L) \cdot \text{LOG}(0.234 \cdot (ee / D)^{1.11} + 5.42 \cdot Nu \cdot D / Q)}$$

DWSJQ:

$$Q = -2.22 \sqrt{(g \cdot D^5 \cdot hf / L) \cdot \text{LOG}(0.27 \cdot ee / D + 4.62 \cdot (Nu \cdot D / Q)^{0.9})}$$

Similar expressions can be stored in variables within TI-89 or TI-92 calculators and solved using the numerical solvers available in those calculators. Notice that in the calculators *hf* represents the head loss h_f , *ee* represents the absolute roughness e , and *Nu* represents the kinematic viscosity ν .

Solution with HP calculators

The numerical solver in the HP calculators is obtained by using [↵][SOLVE][OK] in the HP 48 G series, or [↵][NUM.SLV][OK] in the HP 49 G, HP 49G+, and HP 48 GII calculators. Unless an equation is already stored in variable *EQ*, you will be prompted to enter an equation in the *Eq:* field. Type equation one of the equations above (must be between quotes), or load an existing equation, then press [ENTER]. The resulting input form will include input fields for the variables *hf*, *ee*, *D*, *Q*, *Nu*, *L*, and *g*. To solve for any of the unknowns, first, enter the values of the six known variables, pressing [OK] after each value entered. Then, using the arrow keys, select the field of the unknown variable, and press [SOLVE]. If the value returned is too large to see directly in the input form, press the [EDIT] soft menu key to see the full value in the stack.

Solution with TI calculators

The numerical solver in the TI calculators is obtained by pressing the [APPS] key, and selecting the option *Numeric Solver*. Type an equation in the *eqn:* field, and press [ENTER], or load an existing equation. The numeric solver screen will now show the equation and a list of variable names (*hf*, *ee*, *d*, *q*, *nu*, *l*, *g*) followed by equal signs. The last item in the list represents the bounds for the solution, with default value `bound = {-1.E14,1.E14}`. Using the arrow keys move from field to field and enter the values of the six known variables. Change the bounds in the last item in the list if need be (e.g., you may require your solution to be a positive number, say, in the interval $[0.0, 100.0]$, thus, you could use `bound = {0.0,100.0}`). Then, move the cursor to the unknown variable, enter an initial guess for the result, and, while keeping the cursor in that position, press *F2-Solve*. The result will be shown at the cursor position. [NOTE: if, in the process of finding a solution with the TI calculator you get a *domain error* message, simply change the initial guess of the solution to a larger or smaller value until a solution is produced].

Summary of solutions

The following table summarizes the solution of the three sample problems using numerical solutions of the equations DWCWQ, DWHAQ, and DWSJQ in the HP calculators:

Table 3. Solutions to sample problems with equations (13) through (15)

	DWCWQ(13)	DWHAQ(14)	DWSJQ(15)
Problem [1]	$hf = 8.86 \text{ ft}$	$hf = 8.73 \text{ ft}$	$hf = 8.90 \text{ ft}$
Problem [2]	$Q = 2.69 \text{ cfs}$	$Q = 2.70 \text{ cfs}$	$Q = 2.68 \text{ cfs}$
Problem [3]	$D = 0.928 \text{ ft}$	$D = 0.927 \text{ ft}$	$D = 0.929 \text{ ft}$

4 - Numerical solution of friction problems using functions *DARCY*, *fHA* and *fSJ*

We can use the calculators' numerical solver to solve friction loss problems when the Darcy-Weisbach equation is written in terms of functions *DARCY* (HP calculators only), *fHA*, and *fSJ*. For example, in the HP calculators, the Darcy-Weisbach equation given in (1) in terms of velocity V can be entered in any of the following forms if using functions *DARCY*, *fHA*, or *fSJ* for the friction factor:

$$hf = \text{DARCY}(ee/D, V*D/\text{Nu}) * L/D * V^2 / (2*g) \quad \text{DWfDV(16)}$$

$$hf = \text{fHA}(ee/D, V*D/\text{Nu}) * L/D * V^2 / (2*g) \quad \text{DWfHV(17)}$$

$$hf = \text{fSJ}(ee/D, V*D/\text{Nu}) * L/D * V^2 / (2*g) \quad \text{DWfSV(18)}$$

The names of the variables listed next to the equation numbers above reflect the equation being implemented (*DW* means *Darcy-Weisbach*), the friction factor used (e.g., *fD* means f given by the *DARCY* function), and V means Velocity.

In the TI calculators, we can define the following variables representing the Darcy-Weisbach equation with friction factors given by the Haaland's and Swamee-Jain's equations, respectively:

$$hf = \text{fha}(ee/d, v*d/\text{nu}) * l/d * v^2 / (2*g) \quad \text{dwfhv(19)}$$

$$hf = \text{fsj}(ee/d, v*d/\text{nu}) * l/d * v^2 / (2*g) \quad \text{dwfhv(20)}$$

Notice that variable Nu or nu in these equations represents the kinematic viscosity ν .

[NOTE: Neither the HP nor the TI calculators include the Greek letter ν in their collection of characters.]

Next, we present some examples of numerical solutions of equations (16) through (20):

Example 1

Consider the following data: $Q = 0.05 \text{ m}^3/\text{s}$, $L = 1 \text{ km}$, $D = 0.20 \text{ m}$, $ee = 0.12 \text{ mm}$, $g = 9.806 \text{ m/s}^2$, and $\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}$. The velocity is calculated with equation (2), $V = 4Q/(\pi D^2) = 4 \times 0.05/(\pi \times 0.20^2) = 1.59 \text{ m/s}$. With the values of V , L , D , e , g , and ν , given above, we find that $hf = 12.04 \text{ m}$ (HP calculator, *DARCY*) or $hf = 12.10 \text{ m}$ (TI calculator, *fsj*).

Example 2

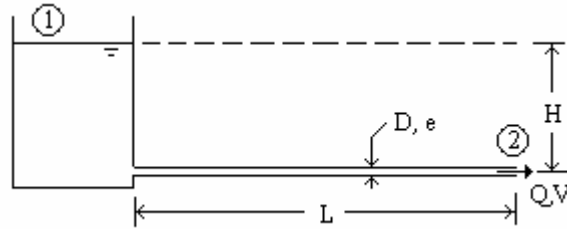
Suppose that we use the same data as in Example 1, above, but we now need to find the diameter required to maintain a velocity of $V = 0.75 \text{ m/s}$ if the head loss in a km of pipeline is to be 5.5 m . Thus, we keep all the values used above, except for $V = 0.75$, $h_f = 5.5$, and solve for D . The result is $D = 0.1174 \text{ m}$ (HP calculator, *DARCY*) or $D = 0.1179 \text{ m}$ (TI calculator, *fsj*).

Example 3

Using the same data as in Example 1, we now determine the velocity in a 0.25-m -diameter, 500-m -long pipe, if the require friction loss is 2.5 m , i.e., use $D = 0.25$, $L = 500$, $h_f = 2.5$. The result is $V = 1.16 \text{ m/s}$ in either the HP calculator (*DARCY*) or TI calculator (*fsj*).

Example 4 – Reservoir-pipe system

Consider the case of a reservoir whose free surface is located at an elevation $z_1 = 60 \text{ m}$, draining through a pipe open to the atmosphere whose outlet is located at an elevation $z_2 = 40 \text{ m}$. The system is depicted in the following figure.



Point 1 in the energy equation is at the reservoir free surface where $p_1 = 0$ and $V_1 = 0$. Point 2, on the other hand, is at the pipe outlet where $p_2 = 0$ and $V_2 = V$, the pipe velocity. In order to make the problem as general as possible, we'll write $z_1 = z_2 + H$, i.e., $H = z_1 - z_2 = 60 \text{ m} - 40 \text{ m} = 20 \text{ m}$, for this case. Writing out the energy equation between points 1 and 2, thus, we find:

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_f,$$
$$z_2 + H + \frac{0}{\gamma} + \frac{0^2}{2g} = z_2 + \frac{0}{\gamma} + \frac{V^2}{2g} + f \cdot \frac{L}{D} \cdot \frac{V^2}{2g},$$

which simplifies to

$$H = \frac{V^2}{2g} \left(1 + f \left(\frac{e}{D}, \frac{VD}{\nu} \right) \cdot \frac{L}{D} \right). \quad (21)$$

In this equation, we'll use $H = 20 \text{ m}$, $L = 100 \text{ m}$, $e = 0.046 \text{ mm}$, $D = 0.5 \text{ m}$, $g = 9.806 \text{ m/s}^2$, and $\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}$. The problem requires us to find the flow velocity V . Equation (8) can be solved using the numerical solvers in either the HP or the TI calculators, entering the equation as:

$$H = V^2 / (2g) * (1 + \text{DARCY}(ee/D, V*D/\nu) * L/D) \quad (22)$$

in the HP calculators, or as

$$h = v^2 / (2g) * (1 + f_{sj}(ee/d, v*d/\nu) * l/d) \quad (23)$$

in the TI calculators.

Using an approach similar to that of the previous examples, we find that $V = 10.69 \text{ m/s}$ (HP calculator) or $V = 10.68 \text{ m/s}$ (TI calculator).

Solving friction loss problems in terms of flow discharge

Instead of using the flow velocity in the Darcy-Weisbach equation, we can write out the equations in terms of the discharge as shown in equation (1). The corresponding equations to be entered in the calculators are:

$$hf = \text{DARCY}(ee/D, 4*Q/(\pi*\nu*D)) * 8*L*Q^2/(\pi^2*g*D^5) \quad \text{DWfDQ(24)}$$

$$hf = f_{HA}(ee/D, 4*Q/(\pi*\nu*D)) * 8*L*Q^2/(\pi^2*g*D^5) \quad \text{DWfHQ(25)}$$

$$hf = f_{SJ}(ee/D, 4*Q/(\pi*\nu*D)) * 8*L*Q^2/(\pi^2*g*D^5) \quad \text{DWfSQ(26)}$$

in the HP calculators, or, in the TI calculators:

$$hf = f_{ha}(ee/d, 4*q/(\pi*\nu*d)) * 8*l*q^2/(\pi^2*g*d^5) \quad \text{dwfhq(27)}$$

$$hf = f_{sj}(ee/d, 4*q/(\pi*\nu*d)) * 8*l*q^2/(\pi^2*g*d^5) \quad \text{dwfhq(28)}$$

Solutions to sample problems [1] through [3]

The following table summarizes the solution of the three sample problems using numerical solutions of the equations DWfDQ, DWfHQ, and DWfSQ in the HP calculators:

Table 3. Solutions to sample problems with equations (13) through (15)

	DWfDQ(24)	DWfHQ(25)	DWfSQ(26)
Problem [1]	$hf = 8.85 \text{ ft}$	$hf = 8.73 \text{ ft}$	$hf = 8.88 \text{ ft}$
Problem [2]	$Q = 2.69 \text{ cfs}$	$Q = 2.70 \text{ cfs}$	$Q = 2.68 \text{ cfs}$
Problem [3]	$D = 0.928 \text{ ft}$	$D = 0.927 \text{ ft}$	$D = 0.929 \text{ ft}$

Additional solutions in terms of discharge are shown next.

Example 5

Consider the data: $Q = 0.05 \text{ m}^3/\text{s}$, $L = 1 \text{ km}$, $D = 0.20 \text{ m}$, $ee = 0.12 \text{ mm}$, $g = 9.806 \text{ m/s}^2$, and $\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}$. With the equations (14) and (15) given in terms of discharge, there is no need to calculate the velocity. Instead, we program the equations in the corresponding calculators and solve directly for h_f . The result is $h_f = 12.06 \text{ m}$ (HP calculator) or $h_f = 12.12 \text{ m}$ (TI calculator).

Example 6

Suppose that we use the same data as in Example 1, above, but we now need to find the diameter required to maintain a discharge of $Q = 0.10 \text{ m}^3/\text{s}$ if the head loss in a km of pipeline is to be 5.5 m . Thus, we keep all the values used above, except for $Q = 0.10$, $h_f = 5.5$, and solve for D . The result is $D = 0.3035 \text{ m}$ (HP calculator) or $D = 0.3038 \text{ m}$ (TI calculator).

Example 7

Using the same data as in Example 1, we now determine the discharge in a 0.25-m -diameter, 500-m -long pipe, if the required friction loss is 2.5 m , i.e., use $D = 0.25$, $L = 500$, $h_f = 2.5$. The result is $Q = 0.057 \text{ m}^3/\text{s}$ in either the HP calculator or TI calculator.

Example 8 – Reservoir-pipe system

Consider the flow described by Equation (8), presented above. This equation was given in terms of the flow velocity. If we re-write it now in terms of the discharge, we find the following equation:

$$H = \frac{8Q^2}{\pi^2 g D^4} \left(1 + f \left(\frac{e}{D}, \frac{4Q}{\pi \nu D} \right) \cdot \frac{L}{D} \right). \quad (29)$$

In this equation, we'll use $H = 20 \text{ m}$, $L = 100 \text{ m}$, $e = 0.046 \text{ mm}$, $D = 0.5 \text{ m}$, $g = 9.806 \text{ m/s}^2$, and $\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}$. The problem requires us now to find the discharge Q . Equation (16) can be solved using the numerical solvers in either the HP or the TI calculators, entering the equations as:

$$'H = 8 * Q^2 / (\pi^2 * g * D^4) * (1 + \text{DARCY}(e/D, 4 * Q / (\pi * \nu * D)) * L/D)'$$
 (30)

in the HP calculators, or as

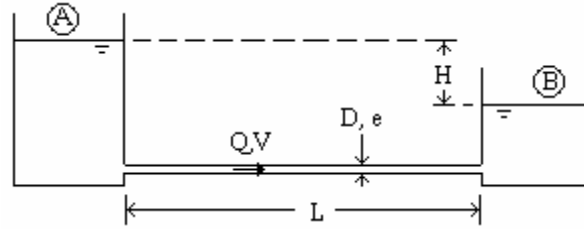
$$h = 8 * q^2 / (\pi^2 * g * d^4) * (1 + \text{sj}(e/d, 4 * q / (\pi * \nu * d)) * l/d) \quad (31)$$

in the TI calculators.

Using an approach similar to that of the previous examples, we find that $Q = 2.10 \text{ m}^3/\text{s}$ (HP calculator) or $Q = 2.10 \text{ m}^3/\text{s}$ (TI calculator).

Example 9 – Two reservoirs connected by a single pipe – case 1

Two reservoirs (A) and (B) are connected by a pipe with $L = 1500 \text{ ft}$, $D = 0.50 \text{ ft}$, and $e = 0.000005 \text{ ft}$. The level of reservoir (A) is maintained at an elevation $H = 20 \text{ ft}$ above that of reservoir (B). Take the kinematic viscosity of water to be $\nu = 1.3 \times 10^{-5} \text{ ft}^2/\text{s}$. Assuming that all minor losses are negligible, except for the discharge loss at the entrance to reservoir (B), calculate the discharge Q . The system is depicted in the following figure.



Setting up the energy equation between the free surfaces of reservoirs (A) and (B), with $p_A = p_B = 0$, $V_A = V_B = 0$, and $z_A = z_B + H$, and including friction losses and discharge losses, the resulting equation turns out to be the same as equation (29). Entering equation (30) into the HP calculators or equation (31) into the TI calculators, with the given data, we find $Q = 1.02 \text{ cfs}$.

Example 10 – Two reservoirs connected by a single pipe – case 2

Suppose that for the system of Example 9 we have $Q = 3 \text{ cfs}$ and we are asked to find D . Using the equation for Example 9 with the same data except for Q , and solving for D results in $D = 0.75 \text{ ft}$.

Example 11 – Two reservoirs connected by a single pipe – case 3

Suppose that for the system of Example 9 we have $Q = 3 \text{ cfs}$ and $D = 0.6 \text{ ft}$, and we are asked to determine the length L of the pipe while all other data remain the same as in Example 9. Using the equation for Example 9 with the same data except for Q and D , and solving for L results in $L = 474.51 \text{ ft}$.

Example 12 – Two reservoirs connected by a single pipe – case 4

Suppose that for the system of Example 9 we have $Q = 2.5 \text{ cfs}$, $D = 0.6 \text{ ft}$, and $L = 1200 \text{ ft}$, and we are asked to determine the elevation H of reservoir (A) over that of reservoir (B) while all other data remain the same as in Example 9. Using the equation for Example 9 with the same data except for Q , D , and L , and solving for H results in $H = 34.31 \text{ ft}$.

5 – Empirical equations for single-pipe flow

For water flow in pipes there are a couple of empirical equations that are often used to calculate flow velocities in pipelines. These are the Hazen-Williams equation and the Manning's equation.

The Hazen-Williams equation

This equation is valid for water in pipes whose diameters are larger than 2 inches and for flow velocities less than 10 fps. The Hazen-Williams equation is given, in BG and SI units, as follows:

$$\text{BG units:} \quad V = 1.318 C_{HW} R_h^{0.63} S^{0.54}, \quad \text{with } V(\text{fps}), R_h(\text{ft}), S (\text{ft/ft}) \quad (32)$$

$$\text{SI units:} \quad V = 1.318 C_{HW} R_h^{0.63} S^{0.54}, \quad \text{with } V(\text{m/s}), R_h(\text{m}), S (\text{m/m}) \quad (32)$$

Where V is the flow velocity, R_h is the hydraulic radius (for a pipe flowing full, $R_h = D/4$), and S is the energy gradient, $S = h_f/L$. The Hazen-Williams coefficient (C_{HW}) depends on the pipe material. Some typical values are:

$C_{HW} = 140$,	smooth straight pipe
$C_{HW} = 110$,	riveted steel or vitrified pipe
$C_{HW} = 90$,	old pipes
$C_{HW} = 80$,	tuberculated pipes

The Manning's equation

The Manning's equation was developed for open-channel flow applications, but can also be used in pipelines. The equation, in BG and SI units, is given as follows:

$$\text{BG units:} \quad V = \frac{1.486}{n_m} \cdot R_h^{2/3} \cdot S^{1/2}, \quad \text{with } V(\text{fps}), R_h(\text{ft}), S (\text{ft/ft}) \quad (32)$$

$$\text{SI units:} \quad V = \frac{1}{n_m} \cdot R_h^{2/3} \cdot S^{1/2}, \quad \text{with } V(\text{m/s}), R_h(\text{m}), S (\text{m/m}) \quad (32)$$

The Manning's resistance coefficient (n_m) depends on the pipe material. Some typical values are:

$n_m = 0.008$,	brass or plastic pipe
$n_m = 0.012$,	concrete pipes
$n_m = 0.014$,	drainage tile, vitrified sewer pipe
$n_m = 0.021\text{-}0.030$	old pipes
$n_m = 0.035$,	tuberculated cast-iron pipes

Non-rigorous head-loss equations

Unlike the Darcy-Weisbach's equation with a friction factor f that depends on the relative roughness (e/D) and the Reynolds number (**R**), the Hazen-Williams and Manning's equations use coefficients that are constant. Since the rigorous derivation of the Darcy-Weisbach equation demonstrated that such should not be the case for pipe flow, the Hazen-Williams and Manning's equations produce non-rigorous, yet practical, head-loss equations.

Using either the Hazen-Williams equations the head loss can be written in the general form:

$$h_f = K \cdot Q^n. \quad (33)$$

Sometimes, to avoid the iterative process typically involved in solving the Darcy-Weisbach equation, it can be assumed that the flow is in the fully-rough regime and that a constant value of f applies. Thus, equation (33) would also apply for this non-rigorous Darcy-Weisbach equation.

The values of K and n for the Darcy-Weisbach, Hazen-Williams, and Manning's equations (the two latter in the BG system only) are given by:

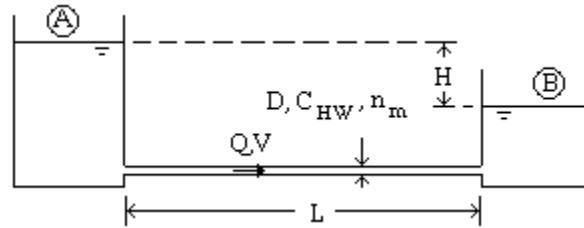
- Darcy-Weisbach:
$$K = \frac{8fL}{\pi^2 g D^5}, n = 2 \quad (34)$$

- Hazen-Williams (BG):
$$K = \frac{4.727L}{C_{HW}^{1.852} \cdot D^{4.87}}, n = 1.857 \quad (35)$$

- Manning's (BG):
$$K = \frac{4.66 \cdot n_m^2 \cdot L}{D^{16/3}}, n = 2 \quad (36)$$

Example 13 – Two reservoirs connected by a single pipe – case 1

Two reservoirs (A) and (B) are connected by a pipe with $L = 1500 \text{ ft}$, and $D = 0.50 \text{ ft}$. The level of reservoir (A) is maintained at an elevation $H = 20 \text{ ft}$ above that of reservoir (B). Use a Hazen-Williams coefficient $C_{HW} = 140$ and a Manning's coefficient $n_m = 0.008$. Assuming that all minor losses are negligible, except for the discharge loss at the entrance to reservoir (B), calculate the discharge Q . The system is depicted in the following figure:



Setting up the energy equation between the free surfaces of reservoirs (A) and (B), with $p_A = p_B = 0$, $V_A = V_B = 0$, and $z_A = z_B + H$, and including friction losses and discharge losses, the resulting equation turns out to be

$$H - h_f - \frac{V^2}{2g} = 0. \quad (37)$$

Replacing the following expression for the velocity head

$$\frac{V^2}{2g} = \frac{1}{2g} \cdot \left(\frac{4Q}{\pi D^2} \right)^2 = \frac{8Q^2}{\pi^2 g D^4} = K_V \cdot Q^2, \quad (38)$$

and equation (33) for the friction loss h_f , equation (37) becomes:

$$H = KQ^n + K_V Q^2 \quad (37)$$

where K is given by either equation (35) or (36), and

$$K_V = \frac{8}{\pi^2 g D^4}. \quad (38)$$

For this problem we find $K_v = 0.403$ and the following values for K and n :

- Hazen-Williams, $K_{HW} = \frac{4.727L}{C_{HW}^{1.852} \cdot D^{4.87}} = 21.98, n_{HW} = 1.857$
- Manning's, $K_m = \frac{4.66 \cdot n_m^2 \cdot L}{D^{16/3}} = 18.04, n_m = 2$

Thus, the system equation (37), for the Hazen-Williams formula is:

$$20 = 21.98Q^{1.857} + 0.403Q^2. \quad (39)$$

A numerical solution with a calculator solver produces the result $\boxed{Q = 0.94 \text{ cfs}}$. If using the Manning's equation, equation (37) becomes

$$20 = 18.04Q^2 + 0.403Q^2, \quad (40)$$

whose solution is $\boxed{Q = 1.04 \text{ cfs}}$.