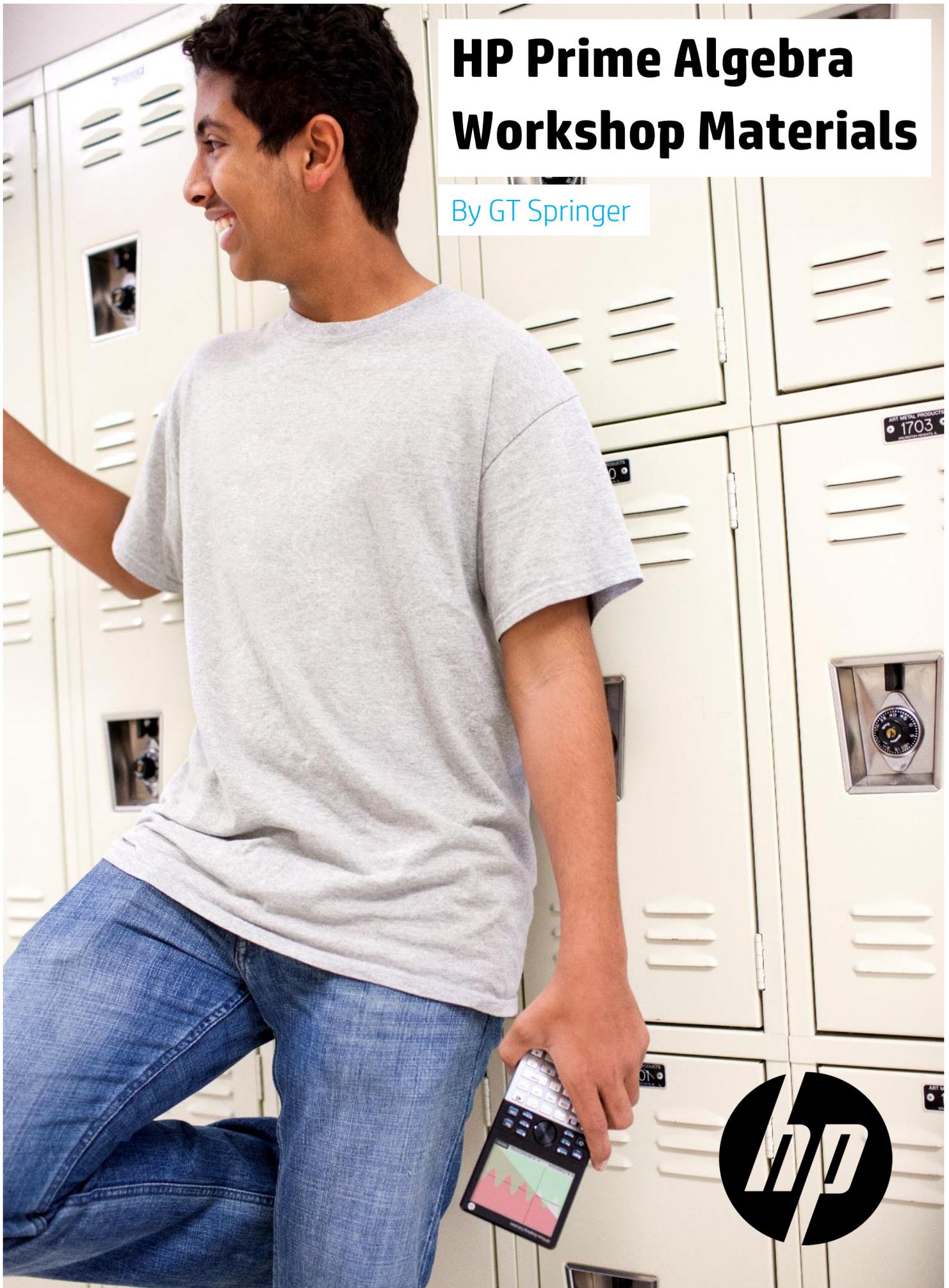# HP Prime Algebra Workshop Materials

By GT Springer

Introduction

This workshop book was designed to introduce teachers of Algebra 1 and Algebra 2 to HP Prime and the HP Prime Wireless Network. The activities in the book are arranged around two powerful themes:
1. The Power of Visualization
2. Looking for Patterns

The Power of Visualization

This section contains activities that help students visualize mathematical concepts or processes. The Advanced Graphing App figures prominently in these activities involving interpretations of graphical representations.

- *Identities and Conditions* uses the Advanced Graphing app to create graphical representations of basic mathematical statements that Algebra 1 students often mistake for identities, such as $\sqrt{X^2} = X$.
- *Special Systems of Linear Equations* examines systems of linear equations of the form ax+by=c, where a, b, and c form an arithmetic sequence in each equation.
- *A Rose for You* is an Algebra 2 activity that involves transformations of a basic graph to create an image.

Looking for Patterns

This section contains activities that encourage students to look for patterns in sequences of numbers, shapes, and even symbolic expressions.

- *An Infinite Series* is an activity for Algebra 2 students that looks at partial sums and the limit of a series.
- *Simplifying Radicals* is an Algebra 1 activity that utilizes the Spreadsheet App and the CAS. In this activity, students look for patterns in the simplification of radicals and write their own rules.
- *Powers of a Binomial* is an Algebra 1 activity that also utilizes the Spreadsheet App and the CAS. It looks at examples of raising the binomial x+1 to various powers and asks students to come up with their own rules for raising x+1 to the nth power.
- *The Sierpinski Triangle* is an Algebra 2 activity that examines the area of the Sierpinski Triangle at various stages and the limit of this area.
- *Solving Trigonometric Equations* is an Algebra 2 activity that merges the power of visualization and the method of looking for patterns into a simple approach to getting to the core ideas in solving equations involving periodic functions.
- *A Peculiar Parabola* is an activity for Algebra 2 students that describes how to use the Advanced Graphing and Spreadsheet apps to decide whether or not the equation $y^2 - 5x = 27$ has any integer solutions. This query came from the marvelous book *Challenging Problems in Algebra* by Alfred S. Posamentier and Charles T. Salking

The Power of Visualization

Graphing calculators are all about graphs and visualization. But the HP Prime graphing calculator, with its Advanced Graphing app, provides new opportunities to visualize mathematical concepts and processes. In the following activities, we look at the power of visualization in a number of topics.

Activity 1: Identities and Conditions

In this activity, we visualize simple mathematical statements and make conjectures involving identities and restricted domains

Visualizing an identity in one variable

What does the graph of an identity look like?

Consider the identity $\dfrac{X+Y}{3} = \dfrac{X}{3} + \dfrac{Y}{3}$.

The graph produced by the Advanced Graphing app is shown to the right. Students must correctly interpret this graph to mean that the statement seems true for all (X, Y) ordered pairs.
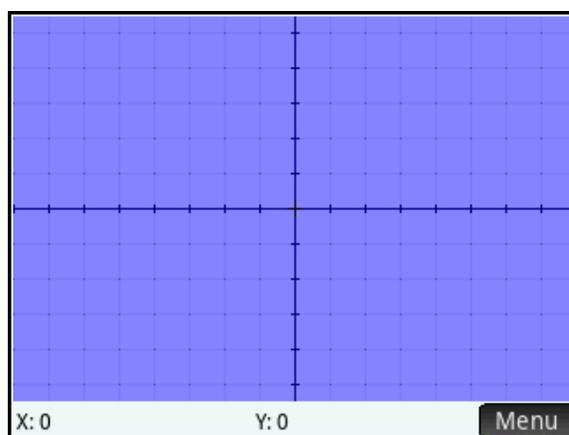

Many students have probably never seen the graph of an identity, so it is worth discussing with them that this graph is saying the equation is true for all (*x*, *y*) ordered pairs. However, by itself, the graph is not a proof of the identity.
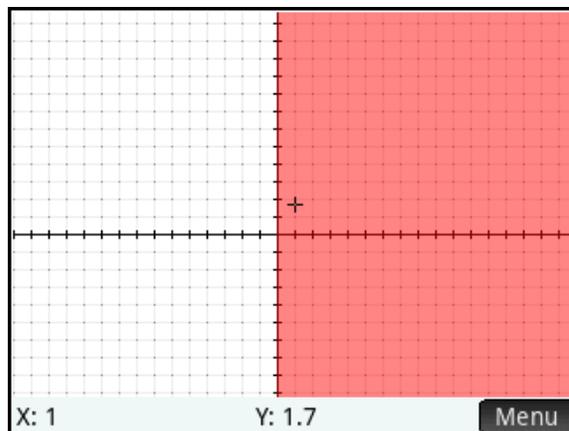
Example 2: Is $|X| = X$ an identity?

1. Press **Apps** and select the Advanced Graphing app
2. Enter $|X| = X$ in V1
3. Press **Plot**

The graph is shown to the right and repeated on the next page. Clearly, the statement is not an identity.
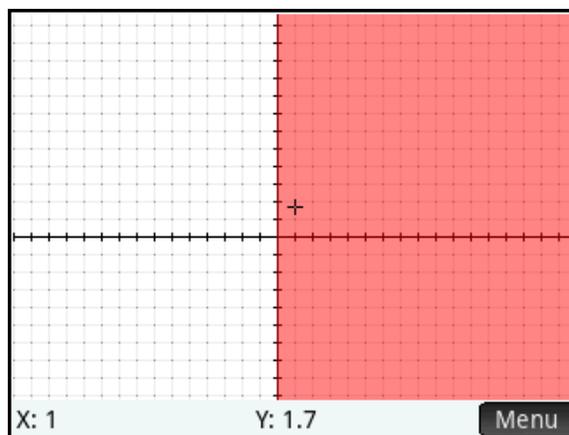
The graph appears to be the graph of X≥0. We entered an equation in V1, but the graph is that of an inequality. This result may be surprising to the students.

4. Press ⬜ to see the Numeric view

The Numeric view (shown to the right) reinforces the notion that the statement is true for X≥0 but false for X<0. We now want to formally qualify the statement, restricting its domain so that it is an identity.
Our conjecture:
If X≥0, then $|X| = X$.

X	Y	V2	
-0.4	-0.4	False	
-0.3	-0.3	False	
-0.2	-0.2	False	
-0.1	-0.1	False	
0	0	True	
0.1	0.1	True	
0.2	0.2	True	
0.3	0.3	True	
0.4	0.4	True	
0.5	0.5	True	
0			

Advanced Graphing Numeric View

At this point in the lesson, we will look at a few more examples.

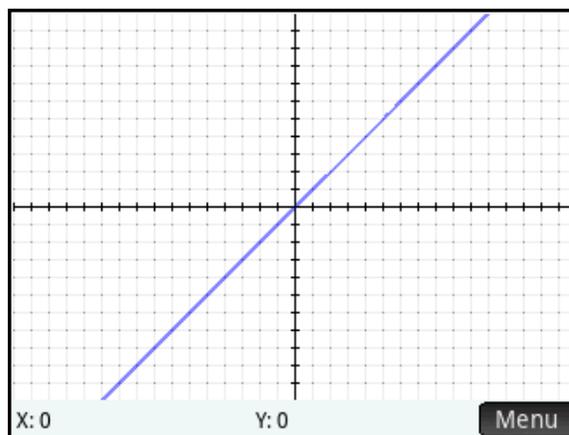Example 3: For what values of X does $\sqrt{X^2} = X$?

Example 4: Is $\sqrt{X^2} = |X|$ an identity?

Now we move on to look at a similar situation involving 2 variables.

Identities in Two Variables

Example 1: Does $X^3 = Y^3$ imply that X=Y?

The graph of $X^3 = Y^3$ is shown to the right. The graph is clearly that of X=Y, so $X^3 = Y^3$ does imply that X=Y.

Example 2:

If $X^2 = Y^2$ does it follow that *X=Y*?

1. Press **Apps Info** and select the Advanced Graphing app
2. Enter $X^2 = Y^2$ in V1
3. Press **Plot→Setup**. The graph is shown to the right.

It seems like there are 2 possibilities and only one of them is that X=Y. Students must identify the other branch as X= -Y.

If $X^2 = Y^2$ then either *X=Y* or *X= -Y*.
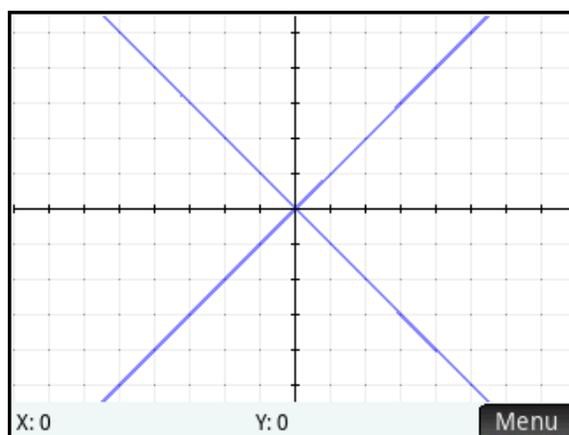
At this point in the lesson, we look at a few more examples.

Example 3: Is $\sqrt{X^2 + Y^2} = X + Y$ and identity? Is it ever true?

Example 4: If $|X| = |Y|$, does it follow that X=Y?

Activity 2: Special Systems of Linear Equations

Suppose you have a set of linear equations in Standard Form: Ax + By = C. Suppose further that the coefficients of each equation (A, B, and C) form an arithmetic sequence. What surprising property do they all have in common? This activity was featured at an NCTM National Conference lesson study presentation by Dan Kennedy.
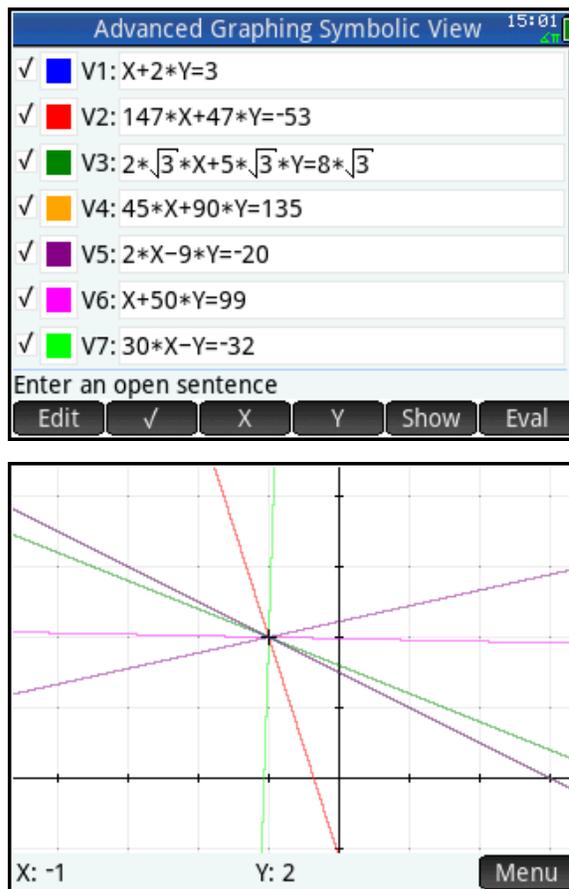
1. Press ⬛ to open the App Library and select the `Advanced Graphing` app.

2. Create several linear equations, each of whose coefficients form an arithmetic sequence. The figure to the right shows a varied set of 7 such equations.

I like to challenge students to come up with their own outlandish examples and use them!

3. Press ⬛ to see the graphs.

It is surprising the first time one sees that these lines are all concurrent at the point whose coordinates are (–1, 2)! Students will be curious about the mathematics that shows why this must be true!

How do we re-write the equation AX+BY=C so that it reflects the fact that A, B, and C form an arithmetic sequence?

	Advanced Graphing Symbolic View	15:01
✓ 🟦	V1: X+2*Y=3	
✓ 🟥	V2: 147*X+47*Y=-53	
✓ 🟩	V3: $2*\sqrt{3}*X+5*\sqrt{3}*Y=8*\sqrt{3}$	
✓ 🟧	V4: 45*X+90*Y=135	
✓ 🟪	V5: 2*X−9*Y=-20	
✓ 🟪	V6: X+50*Y=99	
✓ 🟩	V7: 30*X−Y=-32	

Enter an open sentence

| Edit | ✓ | X | Y | Show | Eval |

X: -1   Y: 2   Menu

Start with AX. Then B must be the sum of A and some value. Let's write AX + (A+B)Y. So the constant C must be B greater than this, or A+2B. So now we have AX + (A+B)Y = A+2B.

By inspection, Y=2 in order to have 2B on both sides. That leaves us:

AX+2(A+B)=A+2B or

AX + 2A + 2B = A + 2B

Now we have 2A on the left but only 1A on the right. So X= -1 is the only way to make the equation true. Thus any equation with this property contains the point (-1, 2).

Activity 3: A Rose For You

In this activity, we discuss how to use the HP Prime Advanced Graphing app to teach students about transformations in the plane, including translations and reflections. But instead of a geometric approach, the app allows us to take an algebraic approach. We start with a generic shape, which we then transform in various ways to build a picture of a rose. The HP App Rose4You already has V1 defined as per Steps 1-3 below. It also has a page of notes to get the student started.

If you do not have the HP App Rose4You, then start by defining our generic shape, a filled ellipse, given by $x^2 + y^2 - x \cdot y - 1 < 0$.

1. Press **Apps Info** and tap the *Advanced Graphing* icon
2. Enter the expression above into V1, using ☐ X ☐ and ☐ Y ☐ to help you
3. Tap on the color picker for V1 and choose purple
4. Press **Plot Setup** to see the graph of our ellipse.
5. Press **+ Ans** to zoom in. This is the first "petal" of our rose.

If you have the HP App Rose4You, then Steps 1-3 are already done for you. Also, Press **Shift** **Apps Info** (Info) to see notes on how to translate, reflect, and dilate V1.

A reflection over the vertical line x=-1 is a mapping such that *x→-2-x* and *y→y*. So we define V2 to be V1(-2-x, y), as shown in the figure to the right. We use the same color (purple) for our reflection.
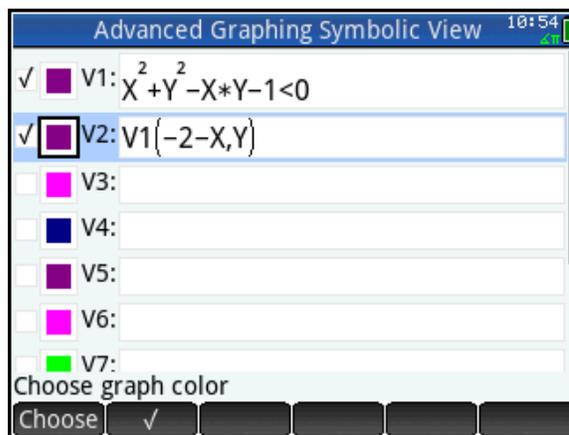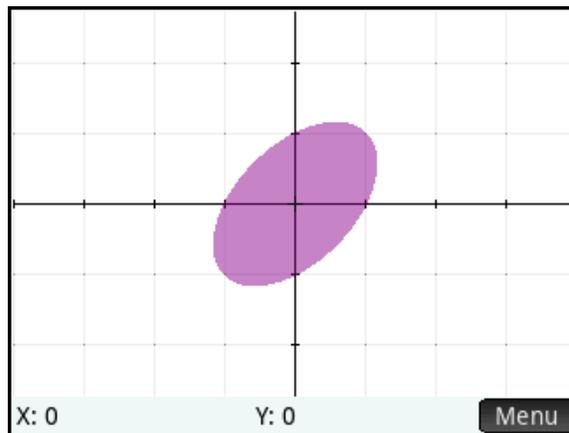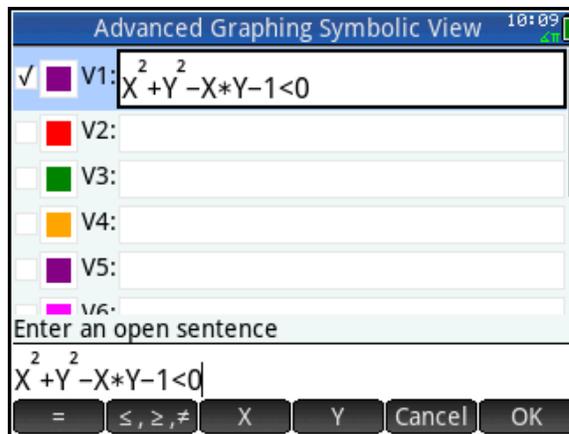
Press ⌨ to see the second "petal"; tap
and drag to scroll the view.
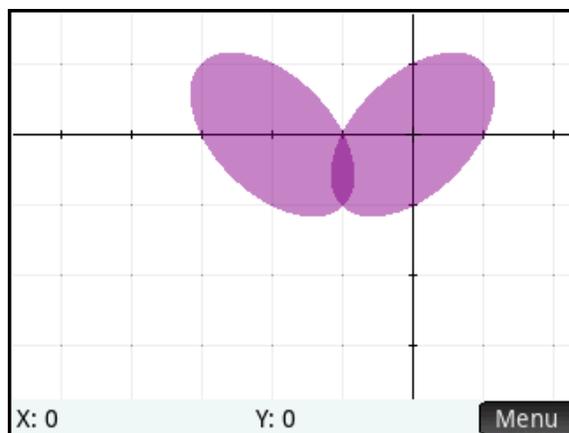
We need a stem for our rose. Creating a
stem takes some experimentation.
Students will learn to stretch the base
shape first, then translate it into its final
position. We defined V3 to be
V1(10*(X+1),0.3*(Y+4)), as shown to the
right. Do not forget to change the color to
dark green!

Again, press ⌨ to see the rose starting to
take shape. Press - to zoom out and drag to
scroll until you get the view you want.
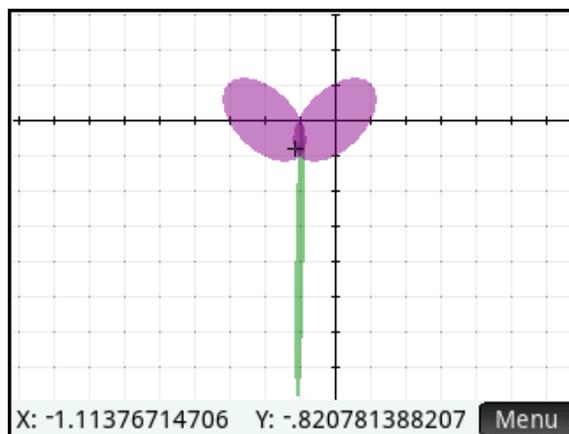
Next, we add some leaves. For the first leaf (in V4), we experiment as we did for the stem, first dilating and then translating to get what we want. Then the second leaf is the reflection of the first leaf, using the same method we used for the petals. We chose light green as the color for both leaves.

And now you have a simple rose!
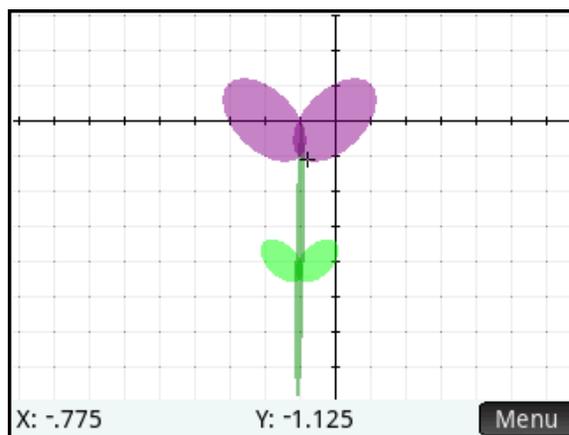
For extra credit, we added two fancy petals and some thorns. The expressions are shown on the next page in case you want to work them out for yourself first!
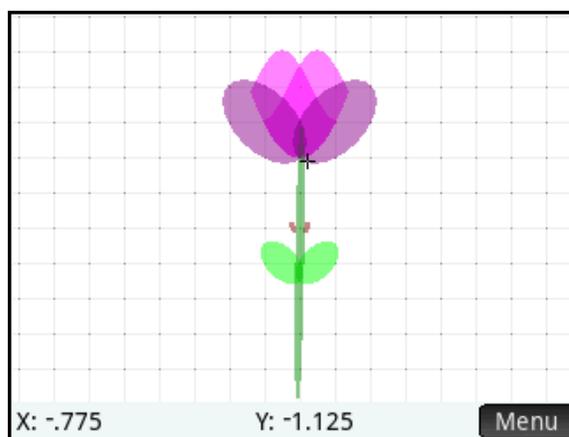
If you did not start with the HP App Rose4You, be sure to save your new app. Press [Apps Info] to go to the App Library; the Advanced Graphing app should already be highlighted. Tap [Save], enter a name for your new app (we used Rose4You), and tap [OK] twice. The new app now appears in your App Library.

Getting the most out of your HP Prime Virtual Calculator for the PC

If you are using the HP Virtual Prime for the PC (it is on the product CD that came with your HP Prime), you can just copy and paste the expressions below into the virtual calculator. Then you can send the app from the virtual calculator to your physical HP Prime. The steps are listed below.

Advanced Graphing app expressions for the rose

V1: X^2+Y^2–X*Y–1<0
V2: V1(–2–X,Y)
V3: V1(10*(X+1),.3*(Y+4))
V4: V1(2*(X+.5),2*(Y+4))
V5: V4(–2–X,Y)
V6: Y+1>(X+1)^2 AND Y<1+COS(2*X+3)
V7: V6(–2–X,Y)
V8: V1(20*(X+.8),10*(Y+3))
V9: V8(–2–X,Y)

To copy the rose expressions to your HP Prime Virtual Calculator

1. Select and copy the expression named V1. Note: select and copy everything <u>after</u> "V1:"
2. On the HP Prime Virtual Calculator, go to the Rose app Symbolic view and select the field **V1**
3. In the menu bar, click on *Edit* and select *Paste*. The expression will be pasted into the V1 field.
4. Tap OK to accept the expression. It will now graph properly.
5. Repeat Steps 2–4 for the other expressions (V2–V9)

To send the Advanced Graphing app from your virtual calculator to a physical HP Prime

1. Connect your physical HP Prime to your PC using the USB cable
2. Turn on your HP Prime and launch the HP Prime Virtual Calculator for the PC
3. On the virtual Prime menu bar, click *Calculator*, then click *Connect To* and select your physical Prime
4. Go to the App Library, select the Rose app, and tap Send
5. The Rose app is now on your physical Prime and ready for viewing!

Looking for Patterns

Looking for patterns in numbers and shapes is a time-honored method for learning mathematics. In this section, we keep to the same tradition, but with the HP Prime CAS, we can extend the method to looking for pattern in symbolic operations.

Activity 1: An Infinite Series

Suppose we wish to explore the series $1 + \dfrac{3}{2} + \dfrac{5}{4} + \dfrac{7}{8} + \dfrac{9}{16} ...$

To begin with, we can compute the first few partial sums.

1. Press [⌂ Settings] to enter the Home view.
2. Type 1 and press [Enter ≈]
3. Press [+ Ans ;] [3 π #] [÷ x⁻¹ T] [2 i Z] [Enter ≈]
4. Press [a b/c ○ ! !! E] to cycle through its decimal, proper fraction, and improper fraction forms
5. Continue as shown to the right until you have the first 5 partial sums

Another approach is to use lists. In the figure to the right, the List L1 is first defined to contain the 1st partial sum (1), followed by the numerator and denominator of the following term. The second line adds the next term to the sum, then adds 2 to the numerator and multiplies the denominator by 2, and stores the result back into list L1. From there, just pressing [Enter ≈] generates the partial sums.

To use the CAS, rewrite the sum symbolically:

$$1 + \frac{3}{2} + \frac{5}{4} + \frac{7}{8} + \frac{9}{16} \ldots = \sum_{n=0}^{\infty} \frac{1+2n}{2^n}$$

1. Press [CAS Settings] to open the CAS view.
2. Press [Units C] to open the template menu and select the summation template
3. To enter n=0, press [ALPHA alpha] [Shift] [( ) N] [▶] 0
4. To enter +∞ at the top, tap on the top of the expression and press [+ Ans ;], then [Shift] [9 S] and tap on ∞
5. Tap on the right template box and enter the rational expression.
6. When you are done, press [Enter ≈]
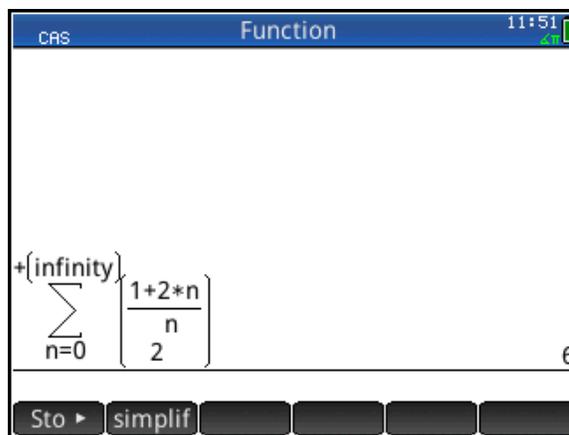
The Spreadsheet app can be used in conjunction with the CAS to enable visualization tools in the symbolic realm, where students can look for patterns and make conjectures about symbolic operations.

Activity 2: Simplifying Radicals

We use a single command to fill the first column with the square roots of the positive integers. The command is simply $\sqrt{Row}$, where *Row* is a variable that refers to the current row of the spreadsheet.

1. Press [Apps Info] to open the Apps Library and tap on the Spreadsheet app icon. The app opens in Numeric view. Tap on the column header for Column A to select the entire column.
2. Press [Shift] [÷] to start a formula. Press [Shift] [x² L] to start a square root. Press [Vars Chars A], tap [App], tap Spreadsheet, Numeric, and select **Row**. Press [Enter ≈] when you are done.
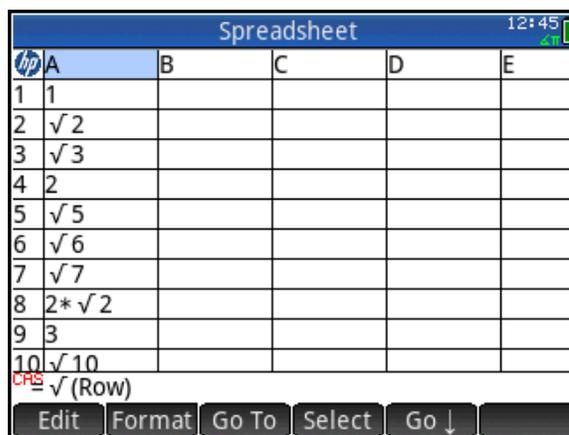
Imagine asking students to look for patterns here! Some integers have square roots that are also integers; others do not. Of the ones that do not, some seem to simplify while others do not. What are the rules here?

Activity 3: Powers of a binomial

The spreadsheet shown below was created with just two commands. Imagine introducing powers of a binomial to students using visualization tools like these! Let's look at the commands used to create this spreadsheet.

The first column contains powers of the binomial x+1, starting at the first power. In fact, the formula for this column is simply $(x+1)^{Row}$, where *Row* is a variable that indicates the row number.

3. Press ⌨Shift ⌨Esc/Clear to clear the Spreadsheet app. You will be prompted to proceed; tap ⌨OK. Tap on the column header for Column A to select the entire column.

4. Press ⌨Shift ⌨± to start our formula. Press ⌨(,)N to enter parentheses. Between the parentheses, enter x+1. To get the lower-case x, press ⌨ALPHA/alpha ⌨Shift ⌨×x. Move outside the parentheses and press ⌨xʸ/F to insert a power. Get the variable *Row* as in Step 2 of the previous example. Press ⌨Enter/≈ when you are done.

5. Let's tidy up a bit. Tap on the HP icon in the upper right corner to select the entire spreadsheet. Tap ⌨Format, scroll down to Textbook, and select Yes. Tap ⌨Format again and select Row↕. In the Row↕ input form, enter 30 and tap ⌨OK. Your Column A should now look like the one in the figure.

6. The second column definition is much like the first, except that it starts with the CAS command *expand.* Tap on the column header for Column B to select the entire column. Press **Shift** [⁝] to start our formula. Press **Mem B** to open the Toolbox menus, tap CAS , Algebra, and select expand. Continue as you did for the first column definition and press [Enter] when you are done.

7. Select one of the rows of expanded binomial powers in Column B and un-pinch (spread) two fingers horizontally to increase the width of the row.

The result is a great visualization of Pascal's Triangle!

Activity 4: The Sierpinski Triangle

The Sierpinski Triangle is a geometric fractal that starts with an equilateral triangle, which we will call our Stage 0. In Stage 1, the midpoints of the sides of the original triangle are joined to form another equilateral triangle, and this triangle is removed from the figure- resulting in three equilateral triangles. Stage 2 continues the process on each of these three new triangles. Stages 1 and 2 are shown below, along with an advanced stage created by the IFSFractal app.

| Stage 1 | Stage 2 | After many iterations... |
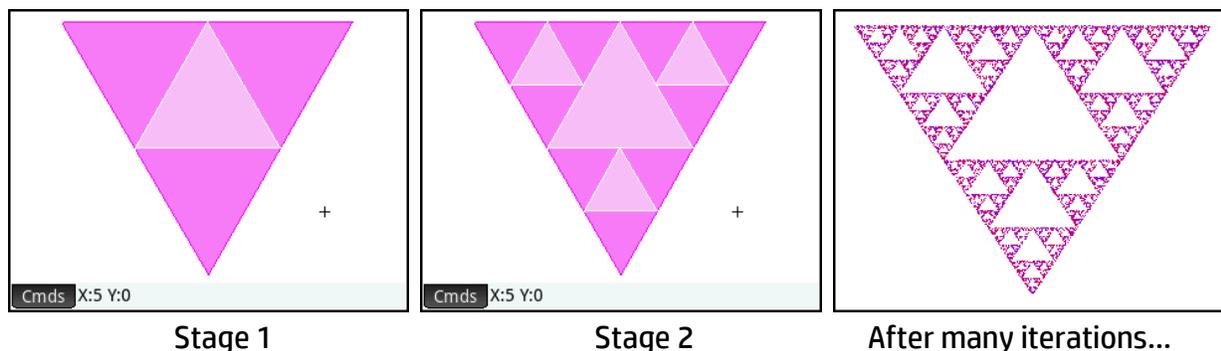
What can we say about the area of the final Sierpinski Triangle? In this activity, we examine the area of the figure at each of the first few stages and then examine its limiting value.

We know that the formula for finding the area of an equilateral triangle whose sides each have length s is $A = \dfrac{s^2 * \sqrt{3}}{4}$. Since we will be using the equilateral triangle area formula repeatedly, we will create a user-defined function based on this formula.

1. Press ⌂ for Home view. Then press **Shift** [x t θ n] (Define) to open the Define input form. For the name, enter TA (for Triangle Area). In the Function field, enter the formula. The field for S appears after the formula is parsed and it will be checked by default to indicate that it will be an input variable for this function. The finished definition appears to the right. Tap OK when you are done.
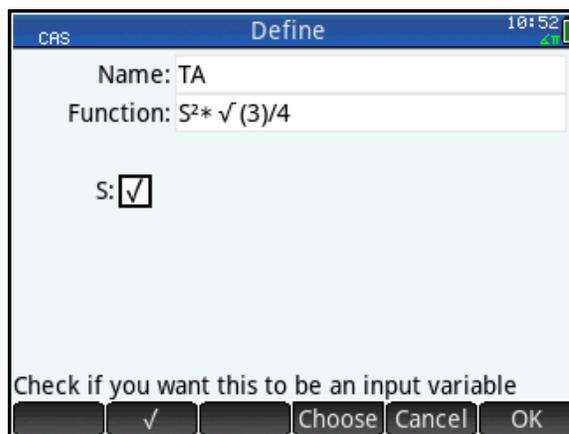
2. Press [Shift] [CAS Settings] (CAS Settings) and set the value of the field Simplify to Maximum. Highlight the field value and tap [Choose] to make a selection, or use [+ Ans] and [- Base] to move through the list of options.

3. Let's test our new user-defined function TA(S). Press [CAS Settings] to open CAS view. Press [Mem B] to open the Toolbox menus. Tap [USER], User Functions, and select TA. The command line will show TA(), with the cursor positioned between the parentheses. Let's assume that the original equilateral triangle at Stage 0 has a side length of 1. Enter 1 between the parentheses and press [Enter ≈].
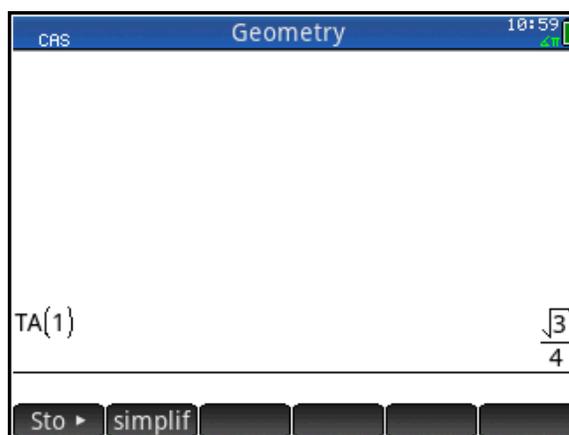
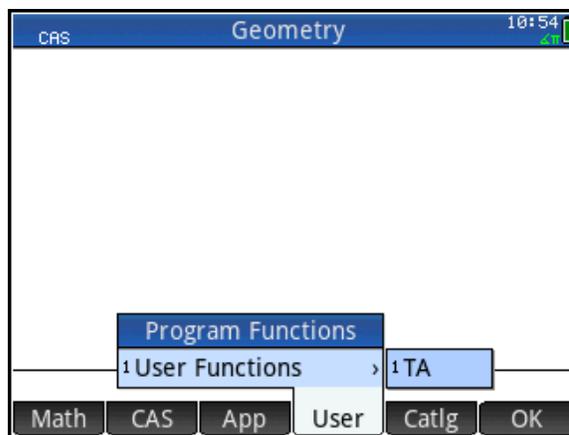The result, $\dfrac{\sqrt{3}}{4}$, is returned simplified as shown in the figure to the right. In Stage 1, we subtract the area of the central triangle, whose sides are each of length 1/2.

4. Press [- Base] and the command line will show Ans-. This means that whatever we enter now will be subtracted from the previous answer. Enter TA(1/2) and press [Enter ≈].

The result is $\dfrac{3\sqrt{3}}{16}$.

5. At Stage 2, we need to subtract the area of three triangles, each of which has sides of length 1/4. Tap on the previous input and tap [Copy] to copy it to the command line. Insert 3* in front of TA and change the 2 in 1/2 to a 4 and press [Enter].

6. At Stage 3, we need to subtract the area of 9 triangles, each with side length 1/8. The results are shown to the right.

We note that the area values all have powers of 4 in the denominator and powers of 3 in the numerator. The general formula for the area at Stage n seems to be $A(n) = \dfrac{3^n \sqrt{3}}{4^{n+1}}$ or

$A(n) = \left(\dfrac{3}{4}\right)^n * \dfrac{\sqrt{3}}{4}$. Clearly, this quantity goes to zero as n→∞. Let's see if the CAS agrees.

7. In CAS view, press [Units c] to open the Template menu and select the limit template. Fill out the template as shown to the right. You can find the symbol ∞ by pressing [Shift] [9 s]. As you can see from the figure, the limit is indeed zero.

It is not accurate to say that the area of the Sierpinski Triangle is zero, as it is not a 2-dimensional object. Its Hausdorff dimension has been established to be less than 2 but greater than 1. Such a figure cannot be said to have an area in the first place.

Activity 5: Solving Trigonometric Equations

In this activity, we look at introducing students to the topic of solving trigonometric equations.

As an example, we take the equation $\sin(x) = \dfrac{\sqrt{3}}{2}$. If we use this equation as an example to introduce students to the notion of solving trigonometric equations, what do we want them to notice from the very outset? First and foremost, there are three important aspects to the solution set:

1. There are an infinite number of solutions
2. These solutions are composed of two branches
3. There is a conventional way of expressing these two branches of infinite solutions in a succinct way

1. Press **Apps** and tap on the Advanced Graphing app icon; the app opens in Symbolic view.

Here, you can enter up to 10 equations or inequalities.

2. In V1, enter $\sin(x) = \dfrac{\sqrt{3}}{2}$

3. Press **Plot** to see the graph.
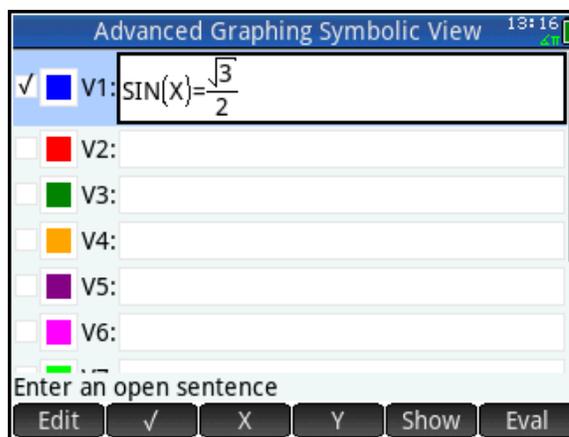
The graph appears to consist of a set of vertical lines. The vertical lines suggest solutions of the form *x=c*, where c is a real number. The lines are arranged in pairs, suggesting two sets of solutions. You can pinch with your fingers to zoom in or out. Zooming out shows more and more of these pairs of lines, suggesting there are an infinite number of solutions. So far, this graphical representation has done a good job of fulfilling our first two requirements.
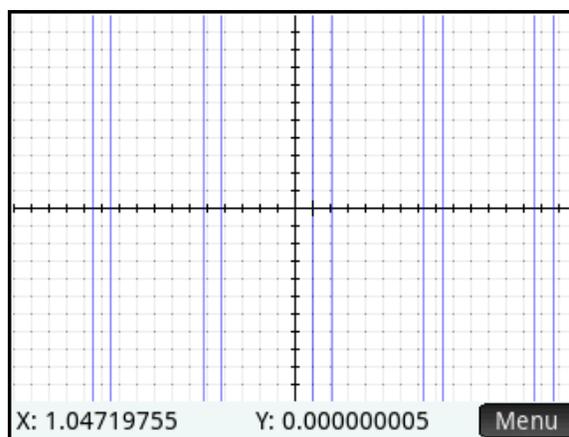
The tracer is on the first vertical line to the right of the y-axis and reads *x*=1.047... or *x*=π/3.

We will now train the tracer to jump from one x-intercept to the next.

Activity 6: A Peculiar Parabola

Does the equation $y^2 - 5x = 27$ have any integer solutions? This problem comes to us from *Challenging Problems in Algebra* by Alfred S. Posamentier and Charles T. Salking (Dover 1996).

1. Press **Apps** to open the App Library and select the `Advanced Graphing` app.

2. The app opens in Symbolic view, where you can enter up to 10 equations or inequalities. In V1, enter our equation. Tap the menu keys at the bottom to enter =, X, and Y. Tap OK or press **Enter** when you are done.

3. Press **Plot** to see the graph

Inspection does not reveal any examples of integer solutions to our equation. Let's use Numeric view to get a closer and more exact sampling of points on this curve.

4. Press **Shift** **Num** to open Numeric Setup view. By default, the X-Step and Y-step values are set to 0.1. Change them both to 1.

5. Press **Num** to open Numeric view. Tap Trace• and select `Edge`.

The table shows the range of integer *y*-values visible in Plot view, and the corresponding *x*-values. You can zoom out in Plot view (press **Plot** **+** or pinch) and then return to Numeric view to see more values. Why would there be no integer solutions to this equation?

We now re-write our equation as $x^2 = 5*y + 27$. The left side of this equation is a square. We now investigate whether or not the right side can also be a square.

6. Press ⬛ to open the App Library and select the `Spreadsheet` app.

7. Tap the column header A to select the entire column. To fill column A with squares, type =Row². Press ⬛ ⬛ to enter the equal sign. Press ⬛, then tap ⬛ and select `Spreadsheet`, `Numeric`, and `Row` to enter the variable *Row*. Press ⬛ to see column A fill with the squares of the positive integers.
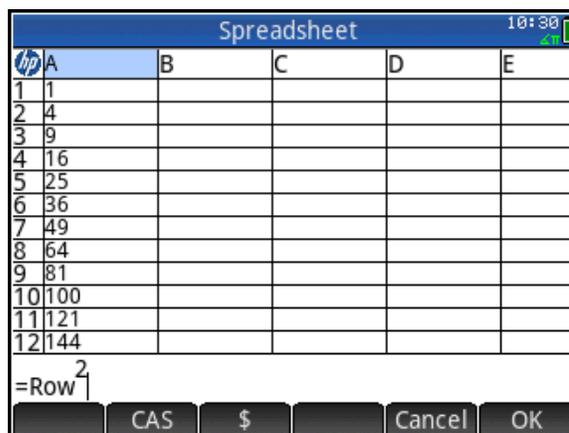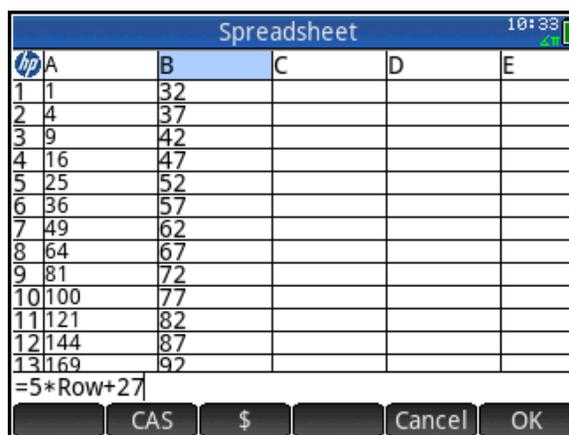
You will notice that the last digits of these squares are from the set {0, 1, 4, 5, 6, 9}.

8. Tap the column header B and enter =5*Row+27.

You will notice that the last digits of the values in column B are from the set {2, 7}. Notice also that these two sets have no common member.

The constant 27 in the formula is not special. This constant could have been any integer of the form $5k+2$.

So now we have an infinite supply of curves with no integer solutions at all! This activity is also a good way to introduce a little number theory into your Algebra classes!

We did not offer a formal proof, but some of your scholars might pursue a more formal explanation of why numbers of the form x^2 cannot also be of the form $5k+2$.

Hint: In order to end in the digits in the set above, the square would have to be in one of the following forms: 5t, 5t+1, or 5t-1. In order to end in 2 or 7, the square would have to be in the form 5t+2.

Challenge: Find another integer n such that 20<n<30, for which the equation $y^2 = 5x + n$ has no integer solution. There are two solutions!