Introduction

There are several applications for Bernoulli Numbers. A very common one is to calculate the sum of powers. For example, if we want to calculate the 15 + 25 + 35 + 45 + 55 + 65 + 75 + 85 = ?

The general formula for this sum is ∑5(x) = 0x -1x2/12 + 0x3 + 5x4/12 + 1x5/2 + 1x6/6

In this example, for x = 8 the sum is 61776

Our task is to determine the coefficients of the polynomial, for a given exponent n.

In the example above, for n = 5, we find a polynomial of degree 6 (always n + 1) and the 6 coefficients are:

B1 = 0, C2 = -1 / 12, C (3) = 0, C (4) = 5/12, C (5) = 1/2 and C (6) = 1/6 (there is no independent term).

The coefficient B1 = 0 is the Bernoulli number for n = 5. From Bernoulli's numbers, we can calculate the remaining coefficients.

See more details in

<https://lucaszanella.com/pt/artigos/mat/bernoulli#:~:text=Ao%20investigar%20a%20f%C3%B3rmula%20para,definidos%20como%20os%20famosos%20n%C3%BAmeros.&text=Cada%20polin%C3%B4mio%20encerra%20com%20seu,sendo%20um%20n%C3%BAmero%20de%20Bernoulli>.

<https://pt.qaz.wiki/wiki/Bernoulli_number>

The program below, separately calculates the numerator and denominator of Bernoulli numbers and coefficients, to give more precision in the result. The values ​​are exact up to n = 27, that is, for Bernoulli Numbers and for the polynomials of the sums of powers from 0 to 27.

The results shown on the calculator screen are stored in the M1 matrix with 4 columns:

In columns 1 and 2 are the numerator and denominator of Bernoulli's Numbers and in columns 3 and 4, the numerator and denominator of the coefficients.

This program was originally written for the HP 39GII calculator which has many commands compatible with the HP Prime.

NOTE: the “J” variable cannot be defined as “LOCAL”, as it is misinterpreted in the “MAKEMAT” command.

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EXPORT BERNOULLI()

BEGIN

LOCAL A,B,C,D,I,K;

LOCAL N,Q;

RECT();

INPUT(K,"Sum of the powers","n=","∑n(x) = 1ⁿ + 2ⁿ + 3ⁿ + 4ⁿ +...+ xⁿ",0);

RECT();

MAKEMAT((J+1) MOD 2,K+1,4)▶M1;

1▶M1(1,1);

1▶M1(K+1,3);K+1▶M1(K+1,4);

IF K>0 THEN

1▶M1(2,1);2▶M1(2,2);

1▶M1(K,3);2▶M1(K,4);

END;

FOR I FROM 3 TO K+1 STEP 2 DO

1▶N;1▶D;

FOR J FROM 2 TO I-1 DO

IF M1(J,1)≠0 THEN

COMB(I,J-1)\*M1(J,1)▶A;

M1(J,2)▶B;

numer(A/B)▶Q;

denom(A/B)▶B;Q▶A;

N\*B+D\*A▶N;D\*B▶D;

numer(N/D)▶Q;

denom(N/D)▶D;Q▶N;

END;

END;

D\*I▶D;D-N▶N;

numer(N/D)▶M1(I,1);

denom(N/D)▶M1(I,2);

M1(I,1)\*COMB(K+1,I-1)▶N;

M1(I,2)\*(K+1)▶D;

numer(N/D)▶M1(K+2-I,3);

denom(N/D)▶M1(K+2-I,4);

END;

16▶C;

FOR I FROM 1 TO K+1 DO

C+1▶C;

IF C>16 THEN

IF I>1 THEN

TEXTOUT\_P("Press any key to continue ...",0,226);

WAIT;

END;

RECT();1▶C;

TEXTOUT\_P("P(x"+K+")=C⑴x+C⑵x²+...+C"+(K+1)+"x^"+(K+1),0,0);

END;

IF M1(I,4)==1 THEN

TEXTOUT\_P("C("+I+")= "+M1(I,3),0,13\*C+5);

ELSE

TEXTOUT\_P("C("+I+")= "+M1(I,3)+"/"+M1(I,4),0,13\*C+5);

END;

END;

TEXTOUT\_P("Press any key to end ...",0,226);

WAIT;

END;

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My best regards,

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