COSTS, RETURN, PROFIT

Program name: CostsReturn v1.01

The program finds for determined preconditions the function of return ***R(x)*** (linear or quadratic function), the function of costs ***C(x)*** (function of 3rd degree: $a*x^3 + b*x^2 + c*x + d$) and calculates characteristic properties concerning both of them and their difference, the profit function: ***P(x)=R(x)-C(x).***
C(x) consists of two parts: ***C(x)*** = $C_x(x)+C_f$, where $C_x(x)$: variable part including terms of x, and C_f (= ***d***): representing the fixed costs.
The following abbreviations are used in the further description:
QU: quantity units, MU: monetary units
COSTS: program variable for ***C(x)***, **RTRN**: variable for ***R(x)***, **PROFIT**: variable for ***P(x) = R(x)-C(x)***
C(x), ***R(x)***, ***P(x)*** as given above

C'(x): marginal costs MU for x QU (≈ increase in costs $\frac{C(x+1)-C(x)}{\Delta x=1}$ ***= ΔC(x)***)
$= 3*a*x^2 + 2*b*x + c$ ≡ tangent to C(x) in point x !
C''(x) = 0: minimum of marginal costs for x QU
$C_v(x) = C(x) - C_f$: variable cost,
$= a*x^3 + b*x^2 + c*x$
c(x) =***((C(x))/(x))***: unit costs
$= \frac{a*x^3+b*x^2+c*x+d}{x}$
c'(x) = ***d(c(x)/dx***
$= 2*a*x+b-\frac{d}{x^2}$
(***c'*** = 0: operating optimum, synonymous with a long-term floor price !)
$c_v(x) = ((C(x)-C_f)/(x)) = a*x^2 + b*x + c$: variable unit cost
$c_v'(x) = d(cv(x)/dx = 2*a*x+b$
(c_v' = 0: operating minimum, identical to a short-term floor price !)
R(x) - C(x) = P(x): profit at x QU
R(x) is defined as ***R(x)=p*x***, where p may be a constant (***R*** in that case is a linear function) or ***p(x) = m*x+n***. In this case ***R(x)*** is a quadratic function in x: $R(x)=(m*x^2+n*x)$.

EXECUTION:

Press The CAS-key. Then uncheck the complex-mode in the CAS-Settings.
To run the program press the toolbox key, then touch the User tab, select CostsReturn and press Enter.
Now the program prompts for the input of R(x). Four choices are possible:

- ***R1.)*** $x_1 = 1$, $R(x_1) = x_2 = R(x_2) = 0$: only costs are analysed, the calculation of **RTRN** is omitted!
- ***R2.)*** $x_2 = R(x_2) = 0$: **RTRN** = $\frac{R(x1)}{x1} * x$, e.g. x_1=3 QU, $R(x_1)$=75 MU → **RTRN = R(x) = 25*x**
- ***R3.)*** $R(x_2) = 0$; x_1 QU are sold for $R(x_1)$ MU and the maximum retail price is at x_2 QU (***R'(x₂)***=**0**)
- ***R4.)*** x_1,$R(x_1)$,x_2,$R(x_2)$ > 0: A linear regression is performed to calculate ***m*** and ***n*** for ***p(x)***.

To enter the cost function directly, input into the fields of the next screen the quantities for ***a***, ***b***, ***c***, ***d***. If you leave the screen as indicated (a=1, b=c=d=0) and press Enter, the next screen prompts for the input of x_0 and ***f(x₀)***. After this entry the following screen is opened to input one of these nine criteria:
(x_0 as QU, $f(x_0)$ as MU):

- ***C1.)*** Costs $C(x_0) = f(x_0)$
- ***C2.)*** Variable costs $C_v(x_0) = f(x_0)$
- ***C3.)*** Marginal costs $C'(x_0) = f(x_0)$
- ***C4.)*** Minimum of marginal costs $C''(x_0) = 0$
- ***C5.)*** Unit costs $c(x_0) = \frac{C(x_0)}{x} = f(x_0)$
- ***C6.)*** Operating optimum $c'(x_0) = \frac{C'(x_0)}{x} = f(x_0)$
- ***C7.)*** Variable unit costs $c_v(x_0) = \frac{C_v(x_0)}{x} = f(x_0)$
- ***C8.)*** Operating minimum $c_v'(x_0) = \frac{C'_v(x_0)}{x} = f(x_0)$

C9.) Profit $P(x_0) = R(x_0)-C(x_0) = f(x_0)$. This criterion enables the user to enter x_0 for the Break-Even-Point or for the upper limit of profit, where $f(x_0) = 0$.

To select the type, scroll to the appropriate line or simply press the concerning key 1 – 9
(default: type ***C1.)*** → Costs).
After the first input of x_0, $f(x_0)$ and type, these entries have to be repeated three times.
Following the calculation the results are displayed, in which the lines with asterisk * are omitted, if no equation for **RTRN** has been calculated (input type ***R1.)***):

- Equation of ***C(x)***,
- equation of ***R(x)***, *
- equation of ***P(x)***, *
- maximum profit P_{max} at (xP) and retail price for P_{max} per unit *.
- breakeven point (***R(x)***=***C(x)***) at x_{BE} and quantity MU, *
- upper limit of intersection ***R(x)***=***C(x)*** at x_{UL} and quantity MU, *
- the operational optimum, where c'=0,
- the operational mnimum, where c_v'=0.

Press the Enter- or CAS-key to depict the graphs of ***C(x)*** (= F1), ***R(x)**** (= F2), ***P(x)*** (= F3)*, ***c(x)*** (= F4) and ***c_v(x)*** (= F5). Possibly you must zoom in to see the function of F4 and F5. In case of criterion ***R1.)*** only Functions F1, F4 and F5 are depicted

EXAMPLE 1:

The HappyLawn Company Inc. plans the introduction of a new robotic lawnmower (**LM**) into the market.
The market research reveals that the product can be sold at a price of 1140€ per unit for 80 sold units, for 135 sold devices the price is maximum.
The cost function has to meet the following criteria:

1.) The fixed costs are 5000 €.	***C1.)***	→	C(0) = ***d*** = 5000
2.) The marginal costs for 80 LM are 75 € per unit.	***C3.)***	→	C′(80) = 75
3.) At the same amount the marginal cost are minimum.	***C4.)***	→	(C′)′(80) = C″(80) =0
4.) Variable unit costs for 20 mowers are 815 €	***C7.)***	→	c_v(20) = 815

Start CostsReturn, then make the following entries according to criterion ***R3.)*** in the screen for R(x) (*fig. 1*):

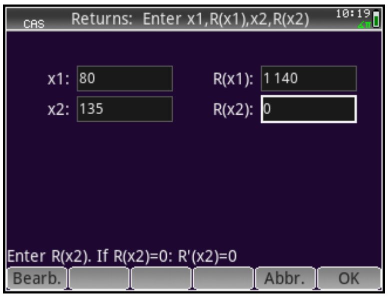

fig. 1

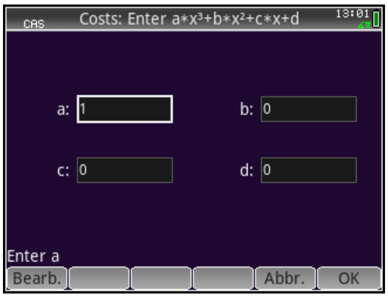

fig. 2

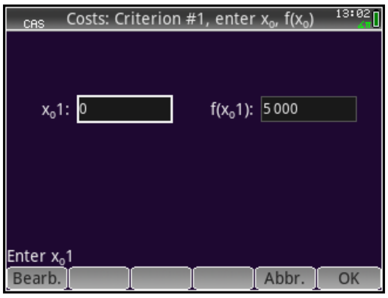

fig. 3

As the function of COSTS has to be calculated, ignore the next screen (*fig. 2*) and press the Enter key.
In the following screen make the inputs for x_0 and $f(x_0)$ according to criterion ***C1.)*** as indicated in (*fig. 3*), press Enter and Enter again (fig. 4 → Type 1: default C(x)).

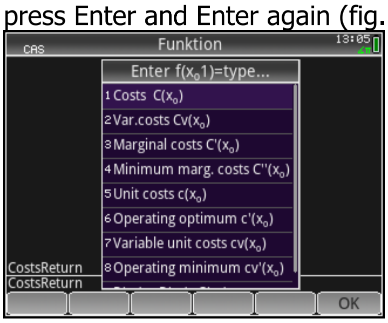

fig. 4

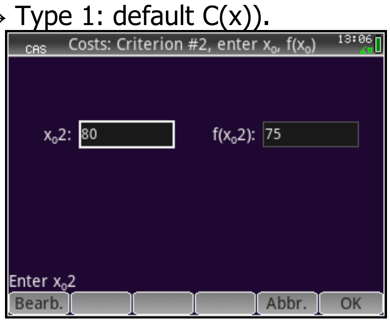

fig. 5

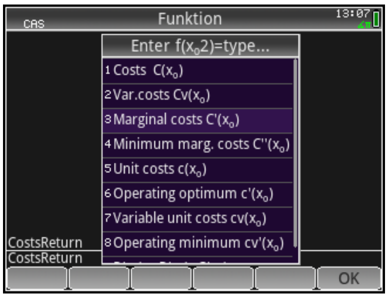

fig. 6

Now complete the input for the remaining conditions ***C3.)*** , ***C4.)*** , ***C7.)*** as demonstrated in *fig. 5 – fig. 10*:

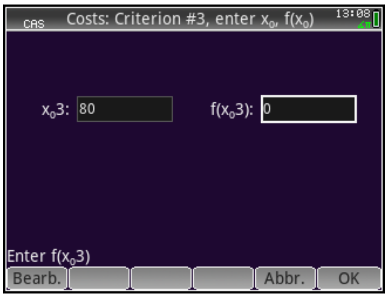

fig. 7

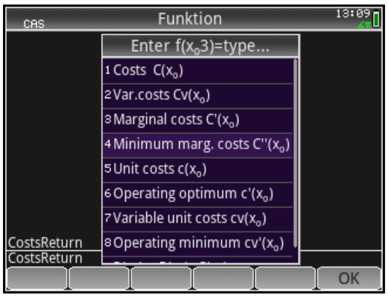

fig. 8

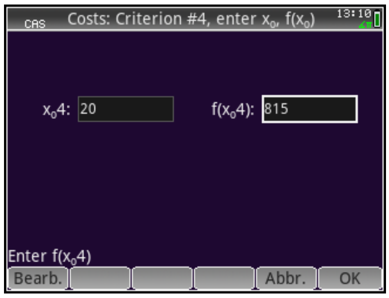

fig. 9

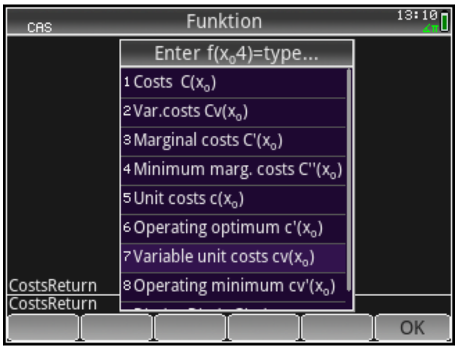

fig. 10

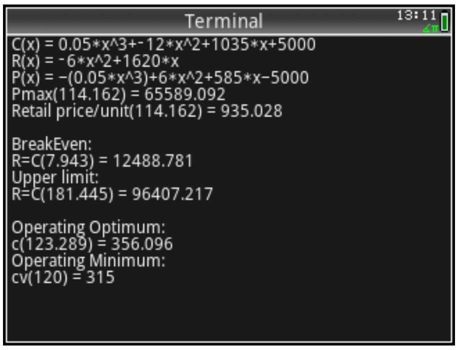

fig. 11

The computed equations are (*fig. 11*):

R(x) = -6*x^2 + 1620*x, ***C(x)*** = $\frac{x^3}{20}$ – 12*x^2 + 1035*x + 5000, ***P(x)*** = $\frac{-x^3}{20}$ + 6*x^2 + 585*x – 5000.

Between ~8 and ~181 sold lawnmowers the company makes profit, in which the maximum profit ***P_{max}*** ~65589 € is made for ~114 devices being sold.

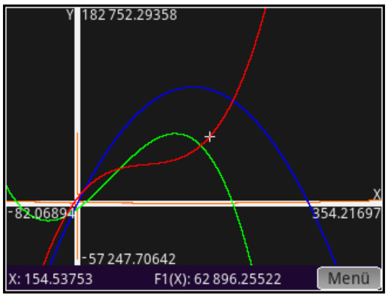

fig. 12

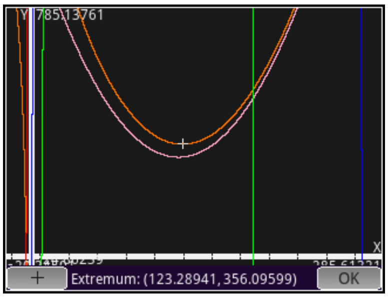

fig. 13

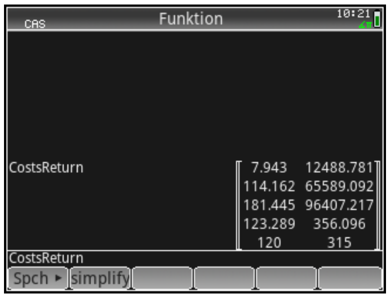

fig. 14

Pressing Enter opens the plot-screen with functions F1 – F5 (*fig. 12*). Zoom in to see F4 and F5 (*fig. 13*). To return to the Home-screen press the CAS key.

Here the numerical results are listed again as matrix: (*fig. 14*)

* Break-Even point:	7.943	12488.781
* ***P_{max}*** :	114.162	65589.092
* Upper limit of costs:	181.445	96407.217
Operating optimum:	123.289	356.096
Operating minimum:	120	315

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