

Differential Equation of 4th and 3rd Order , Part 1

Vers. 1.03

by Claus Martin Dachsel

The program finds the solution $y(x) = y_{hom} + y_{part}$ to the differential equation (D.E.) of 4th order:

$$a*y^{(4)} + b*y''' + c*y'' + d*y' + e*y = g(x)$$

or of 3rd order:

$$b*y''' + c*y'' + d*y' + e*y = g(x), \text{ where } a = 0.$$

The coefficient **a** always must be **1** in a D.E. of 4th order, the same rule applies to **b** in a D.E. of 3rd order.

The right side of the equation may be $g(x) = 0$ (homogeneous D. E.) or one of the following terms making up an inhomogeneous D.E.:

(I) $g(x) = a_4*x^4 + a_3*x^3 + a_2*x^2 + a_1*x + a_0$

(II) $g(x) = f(x)*e^{(\alpha*x)}$, where $f(x)$ may be a polynomial as in (I) or simply a constant.

(III) $g(x) = a*SIN(\alpha*x + \phi) + b*COS(\alpha*x + \phi)$, where **a** and **b** are constants.

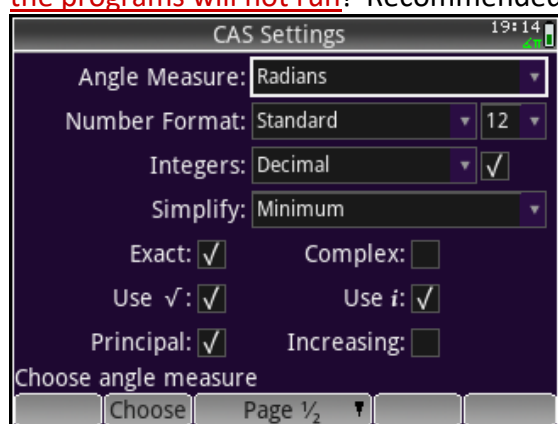
Combinations of (I), (II), (III) can be solved successively, cf. the example below.

Finally, the program can find the numerical values of the constants of integration $C_1 - C_3(C_4)$ for initial conditions $y(x_0)=y_0$, $y'(x_0)=y'_0$, $y''(x_0)=y''_0$, $(y''')(x_0)=y'''_0$.

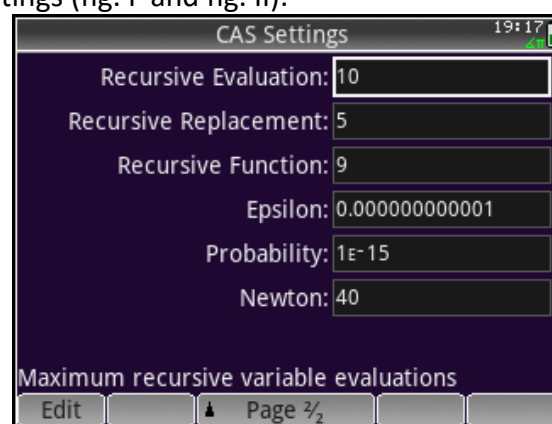
Program execution

The program consists of three parts, which are:

DiffEquat34_h, **DiffEquat34_p**, and **DiffEquat34_num**. These sub-programs have to be transferred to the calculator. Then press the CAS-key to set the calculator to CAS mode, otherwise the programs will not run! Recommended CAS-settings (fig. I and fig. II):



I)



II)

Press the Vars-key, touch the CAS-field and select out of the *programs*-category at first **DiffEquat34_h**, which is obligatory to get the homogeneous part y_{hom} of the solution. It requires the input **DiffEquat34_h(a,b,c,d,e)**. As the variable **e** is reserved internally for Euler's constant, the variable **f** is used instead: **DiffEquat34_h(a,b,c,d,f)**. Enter the quantities and press Enter to get the result, which is displayed in two lines with the D.E. above and the solution y_{hom} in line 2.

To get the solution for $g(x)>0$, program **DiffEquat34_p** is needed.

Here the input is as follows: **DiffEquat34_p(a,b,c,d,f,gx,gx1,gx2)**. You may change the "h" in **DiffEquat34_h** to "p" "manually" and complete the values for **gx,gx1,gx2** or select **DiffEquat34_p** from the Vars – CAS – programs sequence as explained above. **a,b,c,d,f** have the same meaning as in **DiffEquat34_h**, the input of **gx,gx1,gx2** depends on the type of $g(x)$:

(I): The polynomial $g(x)$ is entered in **gx**, while **gx1 = gx2 = 0** (**obligatory!**).

(II): Here **gx** must be input as **0** ! $f(x)$ is an expression in **x** or simply a constant and is entered in **gx1**. The exponent of **e** (only the numerical value α !) has to be input in **gx2**, e.g. an expression like $(3*x^3-2)*e^{-x}$ requires **DiffEquat34_p(.....,0,3*x^3-2,-1)**.

(III): The arguments α and ϕ have to be entered as list in **gx** as $\{\alpha, \phi\}$ and are internally stored in $\{gx(1), gx(2)\}$. The constants **a** in the SIN-term and **b** (COS-expression) are the values to be input in **gx1** and **gx2**, respectively.

Example: $g(x) = 3 \cdot \sin(2 \cdot x + 1) - 4 \cdot \cos(2 \cdot x + 1)$ requires **DiffEquat34_p**(....., {2,1}, 3, -4). A missing term is taken into consideration by 0 for the appropriate factor, e.g.

$g(x) = 7 \cdot \cos(4 \cdot x)$ requires **DiffEquat34_p**(..., {4,0}, 0, 7).

For a combination of different types in **g(x)** the execution of **DiffEquat34_p** must be performed successively. The result again is depicted in a two-line matrix displaying the solution y_{part} for the current **g(x)** in row 1 and the previous results $y_{hom} + \sum y_{part}$ in the bottom line.

Finally, the third sub-program **DiffEquat34_num**(x0,y0,y10,y20,y30) offers the opportunity to determine the numerical values of $C_1 - C_3$ (C_4) for any initial conditions set in advance:

$y(x_0)=y_0$, $y'(x_0)=y_{10}$, $y''(x_0)=y_{20}$, ($y'''(x_0)=y_{30}$). The last term is only regarded if a D.E. of 4th order is treated and will be ignored in a D.E. of 3rd order. The input is carried out as:

DiffEquat34_num(x0,y0,y10,y20,y30). After pressing "Enter" the plot of **y(x)** is depicted in the plot screen. To return to the input screen press the CAS-key to find the numerical values of C_i in line 1 and the complete equation of **y(x)** evaluated in line 2.

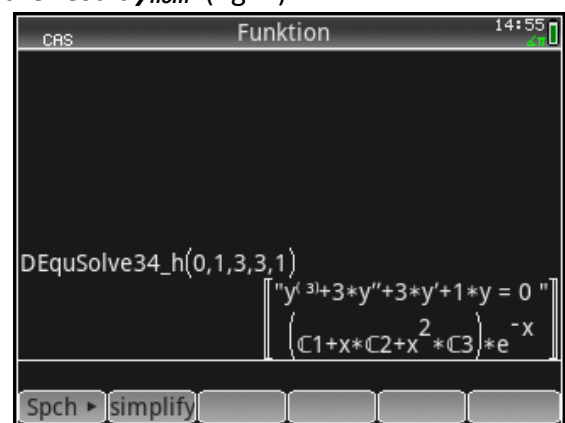
Example

Given the D.E. of 3rd order : $y''' + 3y'' + 3y' + y = x + 6e^{-x} + 8\sin(x)$ with initial conditions: $y(0) = 1$, $y'(0) = -3$, $y''(0) = 0$.

Press the CAS-key to switch to CAS-mode and call up **DiffEquat34_h**. Then enter the coefficients into the brackets (fig.1), press the Enter-key to get the result y_{hom} (fig. 2):

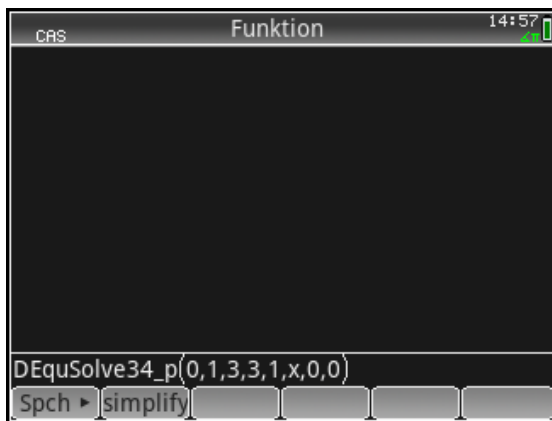


1)

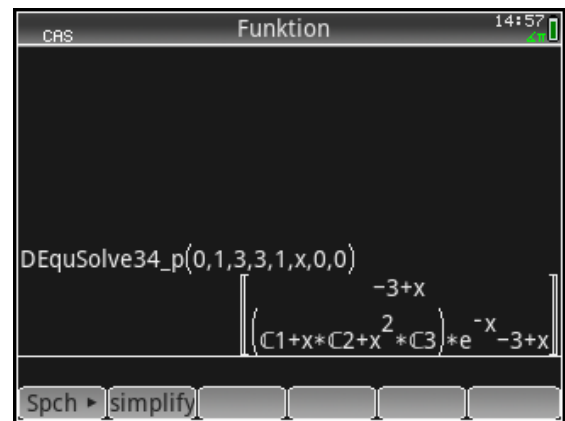


2)

To get the solution for $g(x) = x$ change the input line as displayed in fig. 3 or call up **DiffEquat34_p** and complete the brackets, then press Enter (fig. 4):



3)

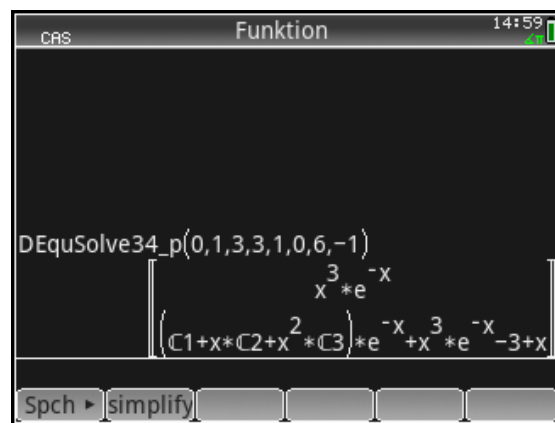


4)

For the next term $g(x) = 6e^{-x}$ to be dealt with change the input line (fig. 5):



5)

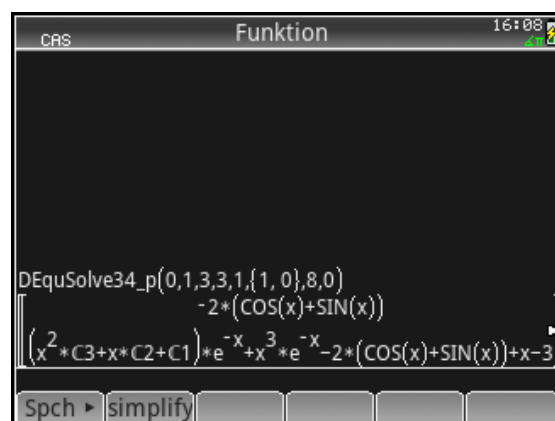


6)

And the last part $g(x) = 8 * SIN(x)$ requires the input as shown in fig. 7:



7)

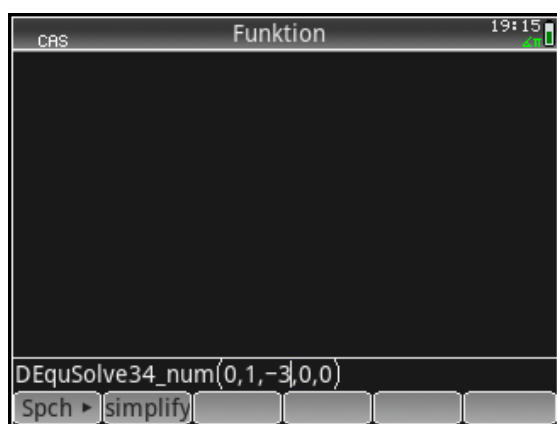


8)

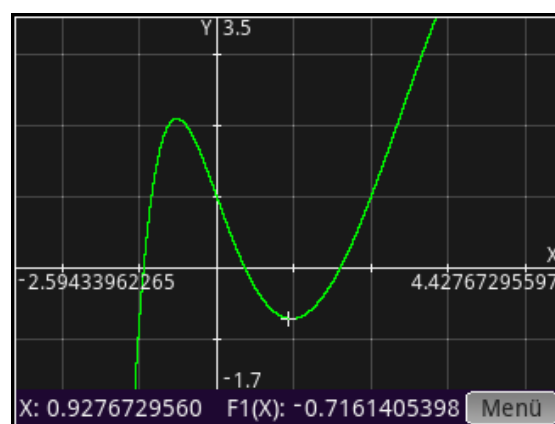
Thus the complete solution to the D.E. is (fig. 8):

$$(C1 + C2 * x + C3 * x^2 + x^3) * e^{-x} - 2 * (SIN(x) + COS(x)) + x - 3.$$

Now call up **DiffEquat34_num(x0,y0,y10,y20,y30)** to find the numerical quantities of C_i and fill out the brackets (fig. 9), Enter-key to get first the plot of the curve (fig. 10):



9)



10)

To return to the input-screen, press the CAS-key. The display depicts the values of C_1 , C_2 , C_3 and the evaluated function $y(x) = y_{hom} + y_{part}$ (fig. 11 and fig. 12.):

CAS Funktion 19:17

```
DEquSolve34_num(0,1,-3,0,0)
C1, C2, C3 = " [[6 4 0]]
"y(x):" 4*x*e-x+x3*e-x-2*COS(x)+6*e-x-2*SIN(x)
DEquSolve34_num(0,1,-3,0,0)
```

Spch ▶ simplify

11)

CAS Funktion 19:18

```
DEquSolve34_num(0,1,-3,0,0)
C3 = " [[6 4 0]]
y):" 4*x*e-x+x3*e-x-2*COS(x)+6*e-x-2*SIN(x)+x-3
DEquSolve34_num(0,1,-3,0,0)
```

Spch ▶ simplify

12)

Reference

L. Papula: Mathematik für Ingenieure und Naturwissenschaftler – Band 2 (8th edition)
Verlag Vieweg (Vieweg Publisher's)