Differential Equation of 4th and 3rd Order , Part 2 *Vers. 1.02*

by Claus Martin Dachselt

This CAS-program is an addition to **Differential Equation of 4th and 3rd Order** and finds the solution $y(x) = y_{hom} + y_{part}$ to the differential equation (D.E.) of 4th order:

$a*y^{(4)} + b*y''' + c*y'' + d*y' + e*y = g(x)$

or of 3rd order ($a = 0$):

$b*y''' + c*y'' + d*y' + e*y = g(x)$, where

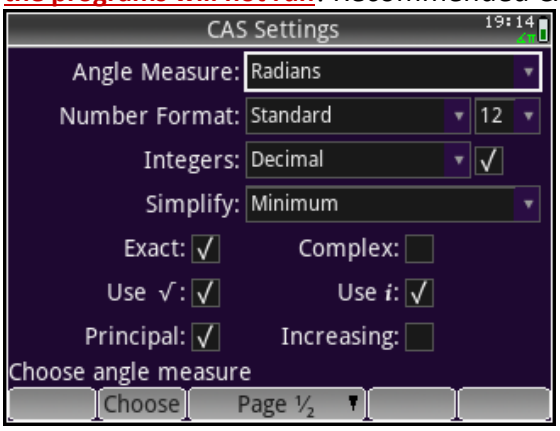
(I) $g(x) = e^{(\alpha * x)} * [c_1*SIN(\beta*x) + c_2*COS(\beta*x)]$, c_1 and c_2 being constants.

The coefficient ***a*** always must be ***1*** in a D.E. of 4th order, the same rule applies to ***b*** in a D.E. of 3rd order.

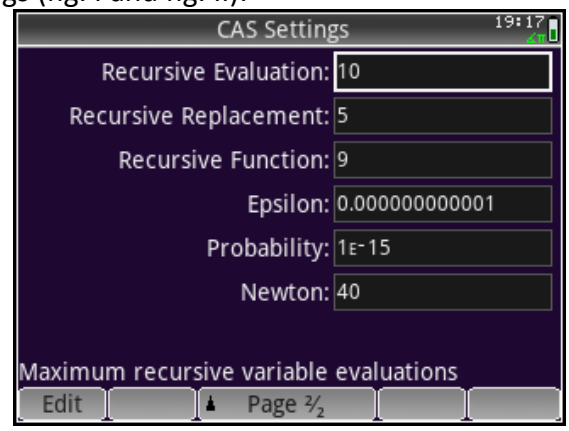
After that, the program can find the numerical values of the constants of integration C_1 – $C_3(C_4)$ for initial conditions $y(x_o)=y_o$, $y'(x_o)=y'_o$, $y''(x_o)=y''_o$, $(y'''(x_o)=y'''_o)$.

Program execution

The program consists of three parts, which are:
DiffEquat34_h2, DiffEquat34_p2, and ***DiffEquat34_num***. The last mentioned is the same sub-program as in **Differential Equation of 4th and 3rd Order, Part 1**. These three programs have to be transferred to the calculator. Then press the CAS-key to set the calculator to CAS mode, **otherwise the programs will not run**! Recommended CAS-settings (fig. I and fig. II):



I)



II)

Press the Vars-key, touch the CAS-field and select at first ***DiffEquat34_h2***, which is obligatory to get the homogeneous part y_{hom} of the solution. It requires the input ***DiffEquat34_h2(a,b,c,d,e)***. As the variable ***e*** is reserved internally for Euler's constant, the variable ***f*** is used instead: ***DiffEquat34_h2(a,b,c,d,f)***. Enter the quantities and press Enter to get the result, which is displayed in two lines with the D.E. above and the solution y_{hom} in line 2.

Now program ***DiffEquat34_p2*** is necessary to get the solution for ***g(x)***.
Here the input is as follows: ***DiffEquat34_p2(a,b,c,d,f,gx,gx1,gx2,gx3)***. You may change the "***h***" in ***DiffEquat34_h2*** to "***p***" "manually" and complete the values for ***gx,gx1,gx2,gx3*** or select ***DiffEquat34_p2*** from the Vars – CAS – programs sequence as explained above. ***a,b,c,d,f*** have the same meaning as in ***DiffEquat34_h2***, the input of ***gx,gx1,gx2,gx3*** , standing for α, β, c_1, c_2 in **(I)**, amounts to the expression:

$g(x) = e^{(\alpha * x)} * [c_1*SIN(\beta*x) + c_2*COS(\beta*x)] \longrightarrow e^{(gx*x)}*[gx2*SIN(gx1*x) + gx3*COS(gx1*x)]$.

For a missing ***SIN***- or ***COS***-term the appropriate factor ***gx2***, ***gx3*** is ***0.*** The factor of $e^{(gx*x)}$ must be ***1***, otherwise a constant factor preceding $e^{(gx*x)}$ must be multiplied by ***gx2*** and ***gx3***:

$4*e^{(gx*x)}*[gx2*SIN(gx1*x) + gx3*COS(gx1*x)] \Rightarrow e^{(gx*x)}*[4*gx2*SIN(gx1*x) + 4*gx3*COS(gx1*x)]$.

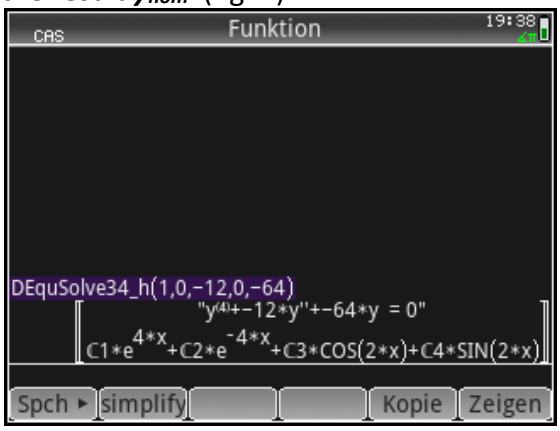
For several different expressions of ***g(x)*** the execution of ***DiffEquat34_p2*** must be performed

successively. The result again is depicted in a two-line matrix displaying the solution y_{part} for the current ***g(x)*** in row 1 and the previous results $y_{hom} + \Sigma y_{part}$ in the bottom line.
Finally, the third sub-program ***DiffEquat34_num(x0,y0,y10,y20,y30)*** offers the opportunity to determine the numerical values of $C_1 - C_3(C_4)$ for any initial conditions set in advance: y(x0)=y0, y'(x0)=y10, y''(x0)=y20, (y'''(x0)=y30). The last term is only regarded if a D.E. of 4th order is treated and will be ignored in a D.E. of 3rd order (***y30=0*** recommended !). The input is carried out as: ***DiffEquat34_num(x0,y0,y10,y20,y30)***. After pressing "Enter" the plot of ***y(x)*** is depicted in the plot screen. To return to the input screen press the CAS-key to find the numerical values of C_i in line 1 and the complete equation of ***y(x)*** evaluated in line 2.

Example

Given the D.E. of 4th order : $y^{(4)} - 12*y'' - 64*y = e^{(-2*x)}*[SIN(2*x) - 2*COS(2*x)]$ and initial conditions: $y(0) = 0,\ y'(0) = 1,\ y''(0) = -2,\ y^{(3)} = 5$.
Press the CAS-key to switch to CAS-mode and call up ***DiffEquat34_h2***. Then enter the coefficients into the brackets (fig. 1), press the Enter-key to get the result y_{hom} (fig. 2):

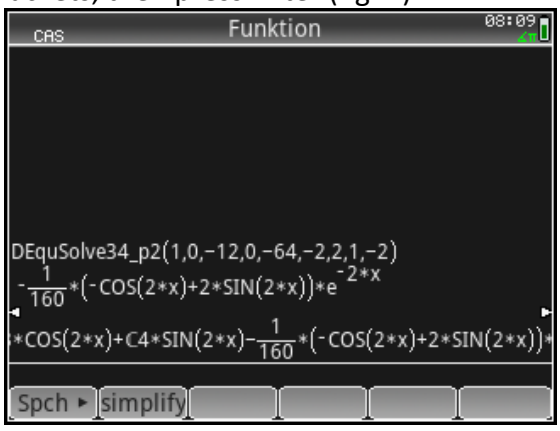
1)



2)

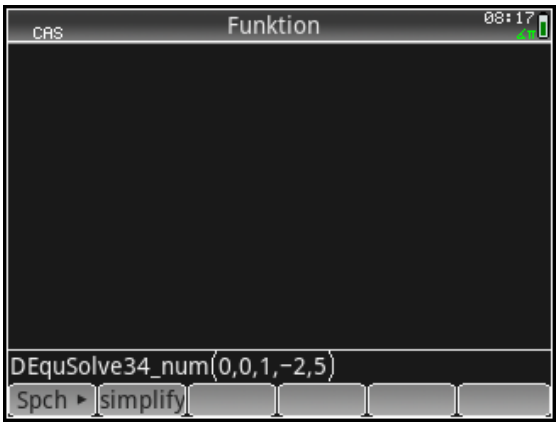
To get the solution for $g(x) = e^{(-2*x)}*(SIN(2*x)-2*COS(2*x))$ change the input line as displayed in fig. 3 or call up ***DiffEquat34_p2*** and complete the brackets, then press Enter (fig. 4):

3)

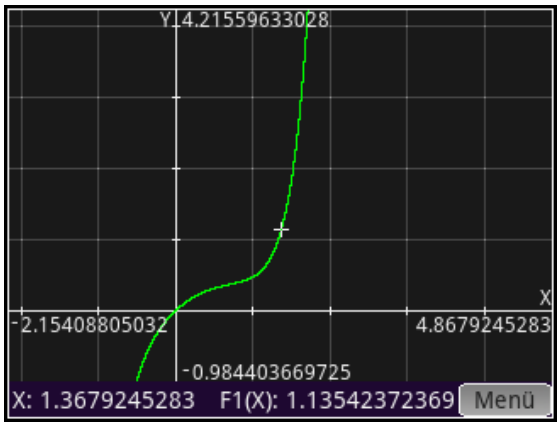


4)

Now call up ***DiffEquat34_num(x0,y0,y10,y20,y30)*** to find the numerical quantities of C_i and fill out the brackets (fig. 5), Enter-key to get first the plot of the curve (fig. 6):

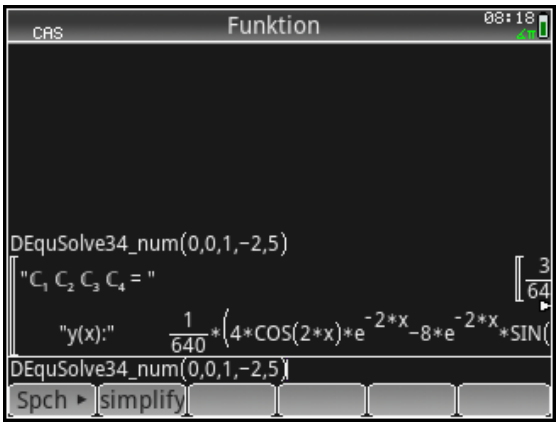


5)

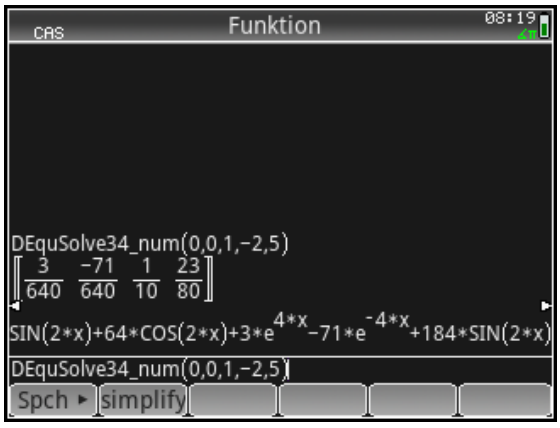


6)

To return to the input-screen, press the CAS-key. The display depicts the values of C_1, C_2, C_3, C_4 and the evaluated function $y(x) = y_{hom} + y_{part}$ (fig. 7 and fig. 8):



7)



8)

For any other initial conditions, ***DiffEquat34_num*** can be repeated with new numerical quantities.