NOLINPACK v1.0

The program NOLINPACK finds the real roots of an equation of the form f(X) = 0. It uses the next iterative techniques: Fixed-Point Method, Newton-Raphson Method, Secant Method, False-Position Method and Bisection Method. This program does not include the Modified Newton-Raphson Method, but you can find it at hpcalc.org as "Modified Newton's Method".

Requirements

The program NOLINPACK was tested and works well in a HP Prime Graphing Calculator with the next characteristics:

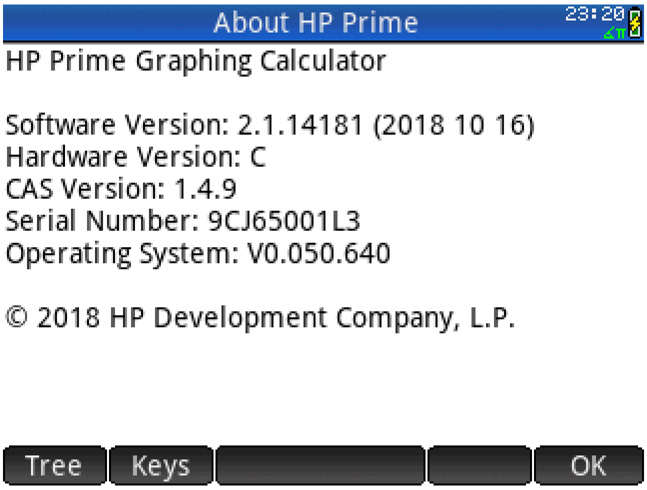


Figure 1. Properties of the HP Prime Graphing Calculator

I don't guarantee the correct functioning of this program in calculators with different software version or CAS version.

Characteristics of the program

- ✓ Run this program in the HOME WINDOW or in the PROGRAM CATALOG WINDOW. DO NOT RUN this program in the CAS WINDOW.
- ✓ Use only 'X' or 'x' as independent variable.
- ✓ This program does not use error or tolerance as a criterion to stop the iterations; instead, the program will show all the iterations you programmed.
- ✓ X0, X1, X2 are the initial approximations o values obtained in the previous iteration.
- ✓ Xr is the approximate value of the root for each iteration.
- ✓ The tolerance or error is calculated as the difference of the values of Xr: ($X_i - X_{i-1}$).

Some considerations to bear in mind:

a. Fixed-Point Method

Verify that g'(x) < 1 to ensure the convergence.

b. Newton-Raphson Method

Verify that the first derivative of f(x) is different from zero; that is, f(X0) ≠ 0. If f(X0) = 0, an error message will display and the program will end.

c. Secant Method

I haven't found any error in the use of this method with this program.

d. Bisection Method and False-Position Method

Input initial approximations X1 and X2, so that f(X1) and f(X2) have opposite signs.

<u>Example:</u>

Find the root of the next equation using iterative techniques:

$$f(x) = e^{-x} - x$$

<u>Solution:</u>

When you run the program, a menu will emerge for you to choose the numerical method you want to use:

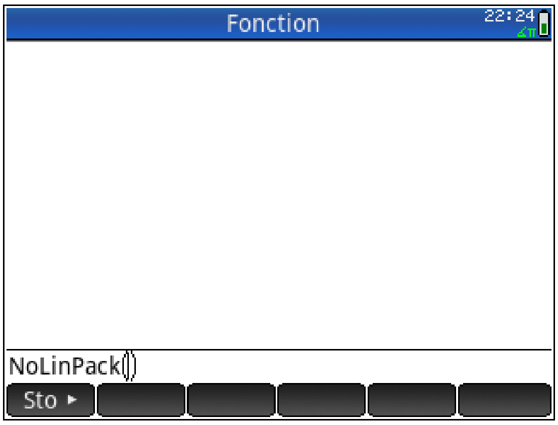


Figure 2. Running NoLinPack

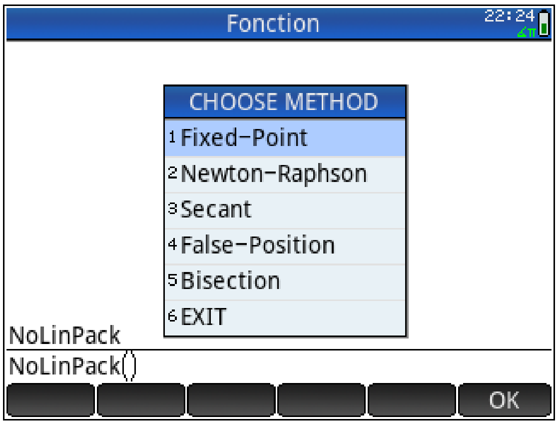


Figure 3. Menu for NoLinPack

After you choose the method, you have to input the necessary data:

- Fixed-Point Method

$$\begin{cases} f(x) = e^{-x} - x \\ g(x) = e^{-x} \\ x_0 = 0 \\ Iterations = 10 \end{cases}$$

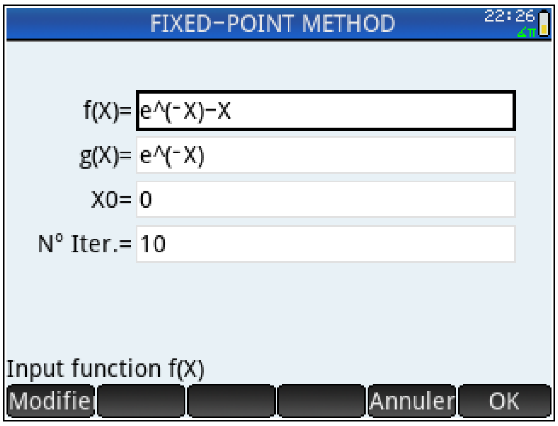
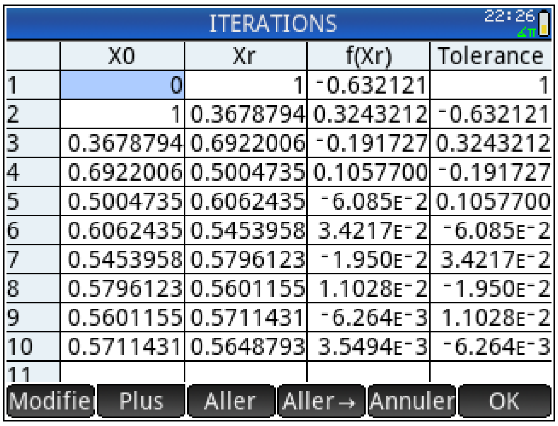


Figure 4. Input window for Fixed-Point Method


ITERATIONS

	X0	Xr	f(Xr)	Tolerance
1	0	1	-0.632121	1
2	1	0.3678794	0.3243212	-0.632121
3	0.3678794	0.6922006	-0.191727	0.3243212
4	0.6922006	0.5004735	0.1057700	-0.191727
5	0.5004735	0.6062435	-6.085E-2	0.1057700
6	0.6062435	0.5453958	3.4217E-2	-6.085E-2
7	0.5453958	0.5796123	-1.950E-2	3.4217E-2
8	0.5796123	0.5601155	1.1028E-2	-1.950E-2
9	0.5601155	0.5711431	-6.264E-3	1.1028E-2
10	0.5711431	0.5648793	3.5494E-3	-6.264E-3
11				

Modifie Plus Aller Aller→ Annuler OK

Figure 5. Iterations using Fixed-Point Method

- Newton-Raphson Method:

$$\begin{cases} f(x) = e^{-x} - x \\ x_0 = 0 \\ Iterations = 4 \end{cases}$$

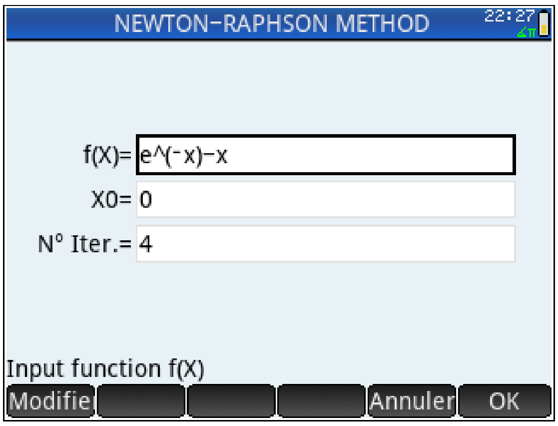
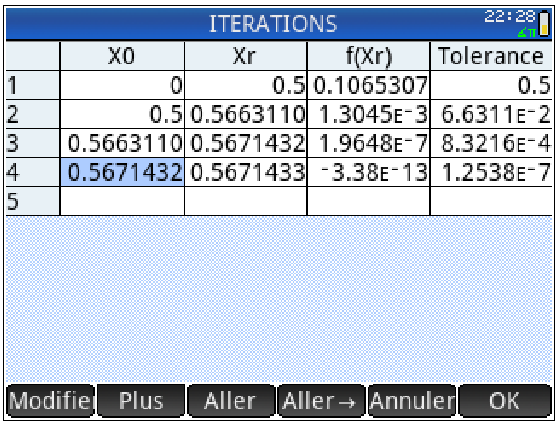


Figure 6. Input window for Newton-Raphson Method


ITERATIONS

	X0	Xr	f(Xr)	Tolerance
1	0	0.5	0.1065307	0.5
2	0.5	0.5663110	1.3045E-3	6.6311E-2
3	0.5663110	0.5671432	1.9648E-7	8.3216E-4
4	0.5671432	0.5671433	-3.38E-13	1.2538E-7
5				

Modifie Plus Aller Aller→ Annuler OK

Figure 7. Iterations using Newton-Raphson Method

- Secant Method:

$$\begin{cases} f(x) = e^{-x} - x \\ x_1 = 0 \\ x_2 = 1 \\ Iterations = 3 \end{cases}$$

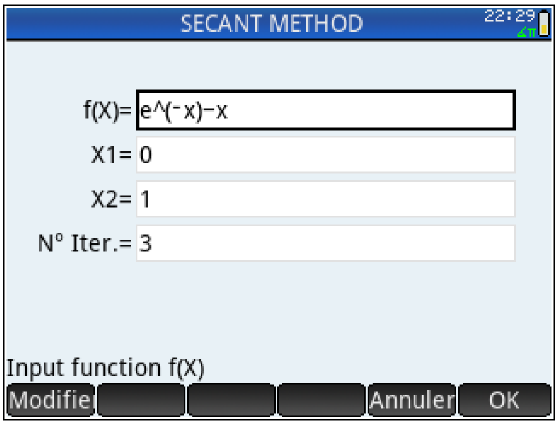
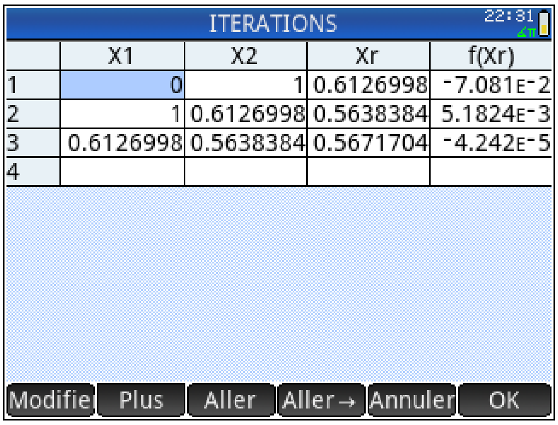


Figure 8. Input window for Secant Method



ITERATIONS

	X1	X2	Xr	f(Xr)
1	0	1	0.6126998	-7.081E-2
2	1	0.6126998	0.5638384	5.1824E-3
3	0.6126998	0.5638384	0.5671704	-4.242E-5
4				

Figure 9. Iterations using Secant Method

- False-Position Method:

$$\begin{cases} f(x) = e^{-x} - x \\ x_1 = 0 \\ x_2 = 1 \\ Iterations = 5 \end{cases}$$

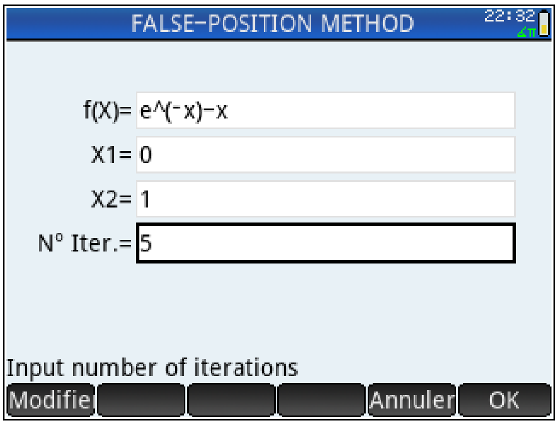
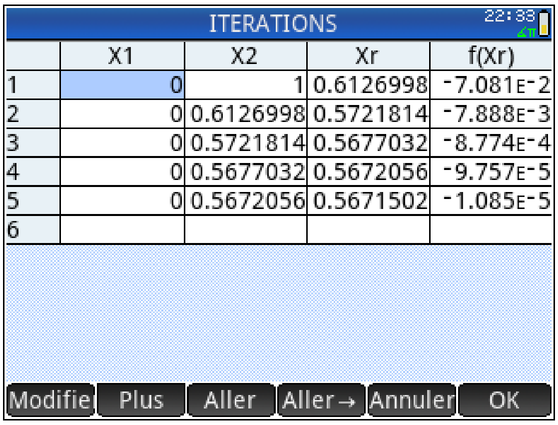


Figure 10. Input window for False-Position Method



ITERATIONS

	X1	X2	Xr	f(Xr)
1	0	1	0.6126998	-7.081E-2
2	0	0.6126998	0.5721814	-7.888E-3
3	0	0.5721814	0.5677032	-8.774E-4
4	0	0.5677032	0.5672056	-9.757E-5
5	0	0.5672056	0.5671502	-1.085E-5
6				

Figure 11. Iterations using False-Position Method

- Bisection Method:

$$\begin{cases} f(x) = e^{-x} - x \\ x_1 = 0 \\ x_2 = 1 \\ Iterations = 10 \end{cases}$$

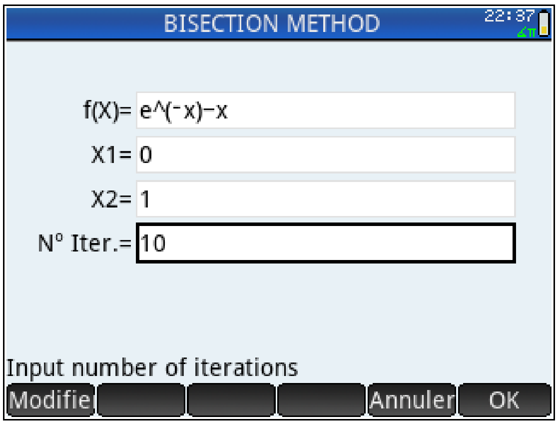
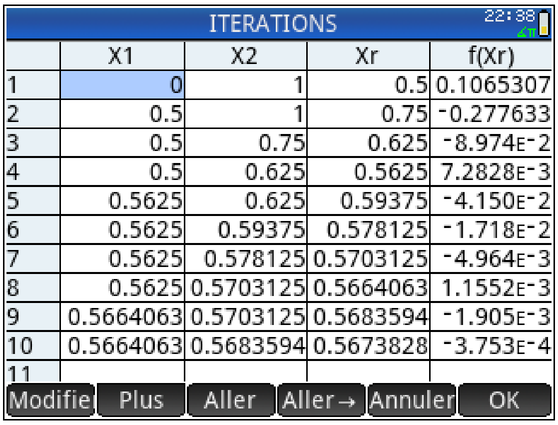


Figure 12. Input window for Bisection Method



ITERATIONS

	X1	X2	Xr	f(Xr)
1	0	1	0.5	0.1065307
2	0.5	1	0.75	-0.277633
3	0.5	0.75	0.625	-8.974E-2
4	0.5	0.625	0.5625	7.2828E-3
5	0.5625	0.625	0.59375	-4.150E-2
6	0.5625	0.59375	0.578125	-1.718E-2
7	0.5625	0.578125	0.5703125	-4.964E-3
8	0.5625	0.5703125	0.5664063	1.1552E-3
9	0.5664063	0.5703125	0.5683594	-1.905E-3
10	0.5664063	0.5683594	0.5673828	-3.753E-4
11				

Figure 13. Iterations using Bisection Method

Author:

Jhonatan Peretz (dj)