

Nwtn4Syst v1.1

The program Nwtn4Syst solves systems of nonlinear equations using the Newton method.  
This program works only in CAS view.

To use this program, write “Nwtn4Syst ()”. This program requires two arguments, a matrix and an integer.  
The matrix must contain three rows:

- First row: system of equations.
- Second row: variables (you can type any variables like x, y, z or x1, x2, x3).
- Third row: initial guess (in the same order you typed the variables).

The next argument is the number of iterations.

Let’s find the roots of the next system of nonlinear equations:

$$\begin{cases} -14 * c1 + 2 * c4 - 70 * c1^{1.2} + 18 = 0 \\ 14 * c1 - 24 * c2 + 2 * c4 - 70 * c2^{1.2} + 9.6 = 0 \\ 24 * c2 - 25 * c3 + c4 - 70 * c3^{1.2} = 0 \\ 25 * c3 - 25 * c4 - 70 * c4^{1.2} = 0 \end{cases}$$

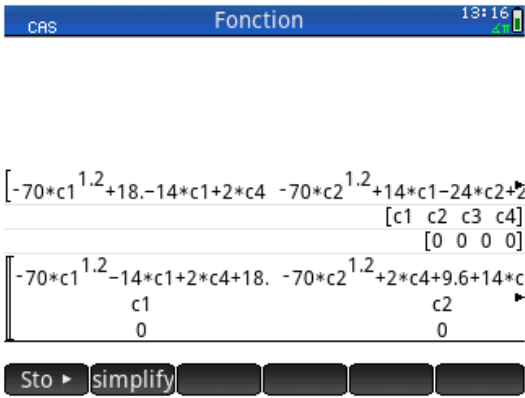
The variables are:

$$\begin{cases} c1 \\ c2 \\ c3 \\ c4 \end{cases}$$

Initial guess:

$$\begin{cases} c1=0 \\ c2=0 \\ c3=0 \\ c4=0 \end{cases}$$

We will use 10 iterations.



So, in our calculator, we type in the name of the program and its arguments, like this:

Nwtn4Syst(

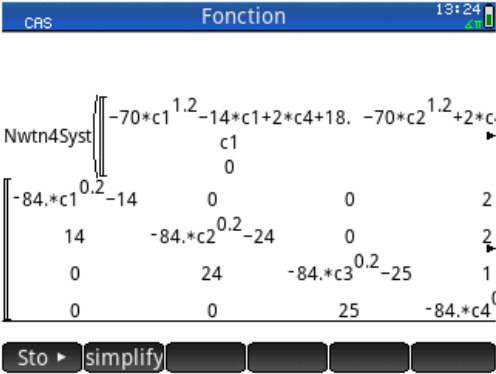
$-70*c1^{1.2}-14*c1+2*c4+18.$	$-70*c2^{1.2}+2*c4+9.6+14*c1-24*c2$	$-70*c3^{1.2}+24*c2-25*c3+c4$	$-70*c4^{1.2}+25*c3-25*c4$
c1	c2	c3	c4
0	0	0	0

,10)

We press ENTER, and the program will show the iterations in a table:

ITERATIONS					
	c1	c2	c3	c4	Tolerance
i:0	0	0	0	0	0
i:1	1.48285714	1.38	1.38	1.38	2.81283937
i:2	0.39023160	0.31813622	0.24849482	0.23403503	2.21696301
i:3	0.27070378	0.18759069	8.112734E-2	5.099596E-2	0.30470347
i:4	0.26659714	0.18155005	6.690628E-2	2.897233E-2	2.721457E-2
i:5	0.26657066	0.18151153	6.670229E-2	2.813534E-2	8.627577E-4
i:6	0.26657062	0.18151149	6.670221E-2	2.813379E-2	1.556516E-6
i:7	0.26657062	0.18151149	6.670221E-2	2.813379E-2	5.38866E-12
i:8	0.26657062	0.18151149	6.670221E-2	2.813379E-2	1.62930E-15
i:9	0.26657062	0.18151149	6.670221E-2	2.813379E-2	1.56719E-15
i:10	0.26657062	0.18151149	6.670221E-2	2.813379E-2	1.57597E-15

Once you press ENTER again, the Jacobian matrix of the system will appear in the CAS view:



The Jacobian matrix is:

$$\begin{bmatrix} -84 \cdot c_1^{0.2} - 14 & 0 & 0 & 2 \\ 14 & -84 \cdot c_2^{0.2} - 24 & 0 & 2 \\ 0 & 24 & -84 \cdot c_3^{0.2} - 25 & 1 \\ 0 & 0 & 25 & -84 \cdot c_4^{0.2} - 25 \end{bmatrix}$$

NEW (v1.1): Error messages

I'll explain this with an example:

Let's try to solve the next system:

Nwtn4Syst  $\left( \begin{bmatrix} -70 \cdot c_1^{0.8} - 14 \cdot c_1 + 2 \cdot c_4 + 18 & -70 \cdot c_2^{0.8} + 2 \cdot c_4 + 9.6 + 14 \cdot c_1 - 24 \cdot c_2 & -70 \cdot c_3^{0.8} + 24 \cdot c_2 - 25 \cdot c_3 + c_4 & -70 \cdot c_4^{0.8} + 25 \cdot c_3 - 25 \cdot c_4 \\ c_1 & c_2 & c_3 & c_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}, 10 \right)$

When we press ENTER, an ERROR message will appear:

CASFunction13:20

Nwtn4Syst

c1

0

...

14\*c1+2\*c4+18

...

-70\*c4<sup>0.8</sup>+25\*c3-25\*c4

c1

c4

0

ERROR at Iteration #1

-70\*c2<sup>0.8</sup>+2\*c4+9.6+14\*c1-24\*c2

-70\*c3<sup>0.8</sup>+24\*c2-25\*c3+c4

-70\*c4<sup>0.8</sup>+25\*c3-25\*c4

±

c3

c4

±

0

0

±

±

±

OK

Next, the table of iterations will be displayed. In this case, the process finished with iteration number 1, so only the initial guess (iteration 0) will be displayed.

Iterations				
	c1	c2	c3	c4
i:0	0	0	0	0
i:1				

EditMoreGo ToGo →CancelOK

Finally, Jacobian matrix is returned.

$$\begin{bmatrix} -14 - \frac{56.}{c_1^{0.2}} & 0 & 0 & 2 \\ 14 & -24 - \frac{56.}{c_2^{0.2}} & 0 & 2 \\ 0 & 24 & -25 - \frac{56.}{c_3^{0.2}} & 1 \\ 0 & 0 & 25 & -25 - \frac{56.}{c_4^{0.2}} \end{bmatrix}$$

OK

With all this data you can analyse what the real problem was. In this example, when we replace the initial guess  $[0 \ 0 \ 0 \ 0]$  in the Jacobian matrix, the elements of the principal diagonal will be  $\pm\infty$ . This means, we cannot continue with the iterations.

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