Aeroassist Orbital Transfer

This MATLAB script can be used to estimate the propulsive ΔV required for aeroassisted coplanar orbital transfer from a high Earth orbit (HEO) to a lower Earth orbit (LEO). Both the initial and final orbits are assumed to be circular. The equations used in this algorithm are described in “Fuel-Optimal Trajectories for Aeroassisted Coplanar Orbital Transfer Problem”, *IEEE Transactions on Aerospace and Electronic Systems,* Vol. 26, No. 2, March 1990, pg. 374-380. Another excellent technical discussion can be found in “Minimum-Fuel Aeroassisted Coplanar Orbit Transfer Using Lift-Modulation”, *AIAA Journal of Guidance, Control and Dynamics,* Vol. 8, No. 1, Jan.-Feb. 1985.

The *normalized* delta-Vs required to initiate the aeropass, $\Delta\tilde{V}_d$, and to re-circularize the orbit after the aeropass, $\Delta\tilde{V}_c$, are given by

$$\Delta\tilde{V}_d = \sqrt{\frac{1}{a_d}} - \sqrt{\frac{2\left(1-a_d\right)}{a_d\left(1-\dfrac{a_d^2}{\cos^2\gamma_{entry}}\right)}}$$

$$\Delta\tilde{V}_c = \sqrt{\frac{1}{a_c}} - \sqrt{\frac{2\left(1-a_c\right)}{a_c\left(1-\dfrac{a_c^2}{\cos^2\gamma_{exit}}\right)}}$$

The *normalized* speeds at entry into the atmosphere, $\tilde{V}_{entry}$, and at exit from the atmosphere, $\tilde{V}_{exit}$, are given by

$$\tilde{V}_{entry} = \sqrt{\frac{2a_d\left(1-a_d\right)}{\cos^2\gamma_{entry} - a_d^2}}$$

$$\tilde{V}_{exit} = \sqrt{\frac{2a_c\left(1-a_c\right)}{\cos^2\gamma_{exit} - a_c^2}}$$

where

$$a_d = \frac{r_d}{r_a} = \text{ initial orbit radius ratio}$$

$$a_c = \frac{r_c}{r_a} = \text{ final orbit radius ratio}$$

$r_d =$ geocentric radius of the initial orbit

$r_c =$ geocentric radius of the final orbit

$r_a =$ geocentric radius of the atmosphere

$\gamma_{entry} =$ flight path angle at atmospheric entry

$\gamma_{exit} =$ flight path angle at atmospheric exit

The dimensional speed and ΔV can be recovered by multiplying the normalized values by $\sqrt{\mu/r_a}$ where μ is the gravitational constant of the Earth. The quantity $\sqrt{\mu/r_a}$ is the local circular velocity at the radius of the atmosphere.

The following is a typical user interaction with this script.

```
                    program aeroassist

< aeroassisted orbit transfer between circular orbits >


please input the initial altitude (kilometers)
? 35786

please input the final altitude (kilometers)
? 300

please input the entry altitude (kilometers)
? 120


please input the entry flight path angle (degrees)
? -3

please input the exit flight path angle (degrees)
? 1
```

The following is the script output for this example.

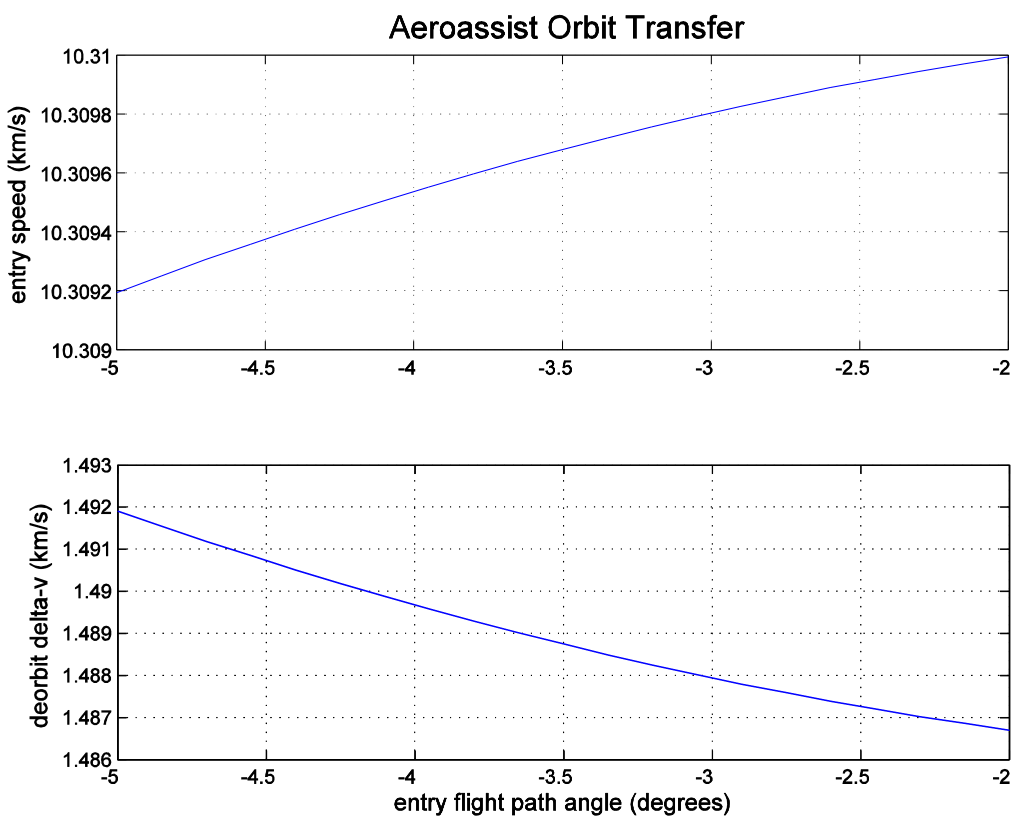
```
aeroassist orbit transfer

initial altitude             35786.0000  kilometers

final altitude                 300.0000  kilometers

entry flight path angle         -3.0000  degrees

entry speed                  10309.8017  meters/second

exit flight path angle           1.0000  degrees

exit speed                    7864.0519  meters/second

deorbit delta-v               1487.9405  meters/second

circularization delta-v         74.8372  meters/second

total delta-v                 1562.7777  meters/second
```

For comparison purposes the software will also display the characteristics of the all-propulsive Hohmann orbit transfer for this example.

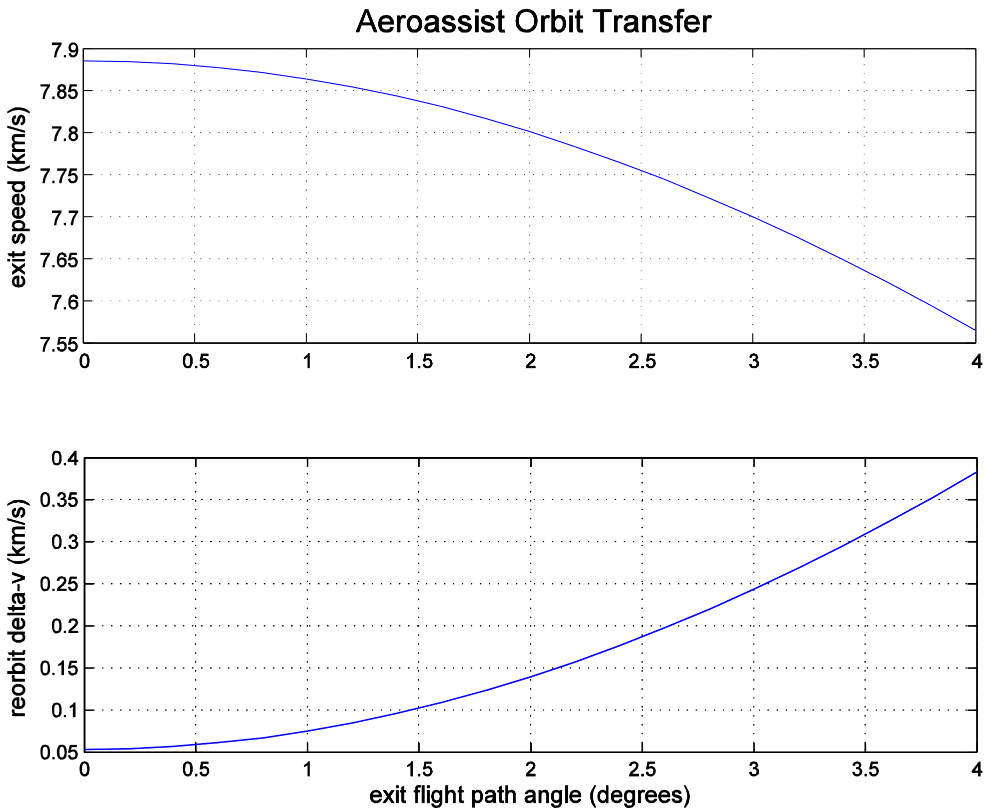
```
Hohmann orbit transfer

deorbit delta-v                1466.8241  meters/second

circularization delta-v        2425.7315  meters/second

total delta-v                  3892.5557  meters/second
```

The `aeroassist` MATLAB script will also create parametric graphic displays of several important flight parameters. In addition to the screen displays, the script will create color `eps` disk files of these plots using source code similar to `print -depsc -tiff -r300 aeroassist3.eps`. These files are created with a TIFF preview.

The following is a plot of the entry speed and de-orbit delta-v as a function of the flight path angle at entry into the Earth's atmosphere.

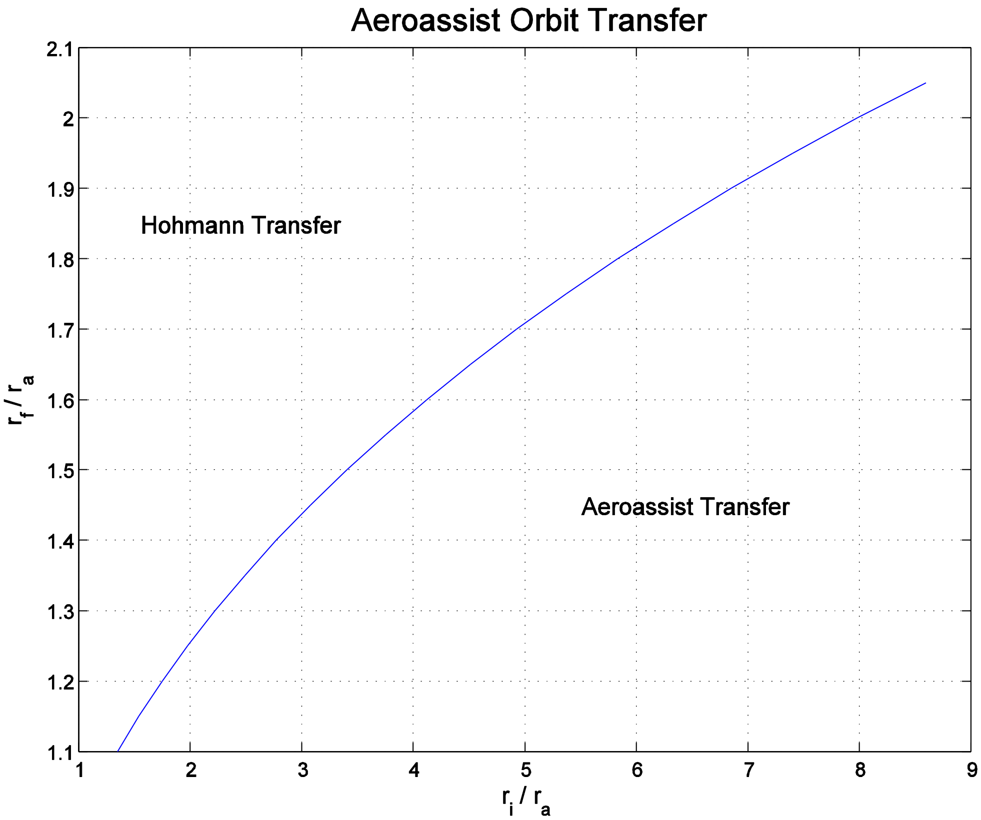


The following is a plot of the exit speed and circularization delta-v as a function of the flight path angle at exit from the atmosphere.



The final plot created by this script illustrates the combinations of initial and final orbit radii where the orbit transfer is performed more efficiently using total propulsive (Hohmann) maneuvers versus aeroassisted maneuvers.

The following is the plot for this example. It uses the atmospheric altitude, entry flight path angle and exit flight path angle provided by the user. In this plot r_i is the radius of the initial circular orbit, r_f the radius of the final circular orbit, and r_a the radius of the atmosphere.


Aeroassist Orbit Transfer
Hohmann Transfer
Aeroassist Transfer
r_f / r_a
r_i / r_a
2.1
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
2
3
4
5
6
7
8
9