Analytic Solution for the Performance of a Single Stage Model Rocket

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* arocket.in                *
* single stage - B4 engine  *
* May 18, 2014              *
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launch site altitude (meters)
100.0

launch site temperature (degrees F)
70.0

rocket engine total impulse (newtons-second)
4.29

rocket engine thrust duration (seconds)
1.03

rocket engine propellant mass (grams)
6.0

initial vehicle mass (grams)
40.0

frontal diameter (millimeters)
18.0

drag coefficient (non-dimensional)
0.321
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Technical discussion

The atmospheric density at the launch site is a function of the site altitude and temperature at the launch time as follows:

$$\rho = \frac{1.22557\,(1 - 2.2556913 \cdot 10^{-5} h)^{4.256116}}{1 + \dfrac{(T - 59)}{518.67}}$$

where

ρ = launch site density (kilograms/cubic meter)

h = launch site altitude (meters)

T = launch site temperature (degrees F)

This equation is used to compensate for "non-standard" launch conditions.

The altitude at rocket engine burnout is given by:

$$X_{bo} = (m + k) \ln\left\{\cosh\left[T_d \sqrt{k(F - mg)} + m\right]\right\}$$

and the burnout velocity is

$$V_{bo} = \sqrt{(F - mg) + k}\,\tanh\left\{T_d \sqrt{k(F - mg)} + m\right\}$$

where

m = average rocket mass (kilograms)

$k = \frac{1}{2}\rho C_d A$

ρ = atmospheric density

C_d = the drag coefficient of the rocket (non-dimensional)

A = cross-sectional area (square meters)

F = average thrust = $\frac{I_t}{T_d}$ (newtons)

I_t = rocket engine total impulse (newton-seconds)

T_d = thrust duration (seconds)

The altitude gained by the rocket during the coasting portion of the flight is given by:

$$X_c = (m + 2k)\ln(kV_{bo}^2 + mg + 1)$$

and the duration of the coasting flight is

$$T_c = \sqrt{m + kg}\,\text{atan}\left\{V_{bo}\sqrt{k + mg}\right\}$$

Finally, the maximum altitude attained by the rocket and total flight time are given by the next two "summation" equations:

$$X_{\max} = X_{bo} + X_c$$

$$T_{total} = T_d + T_c$$

Additional information about the algorithm used in `arocket` and other aspects of rocket flight can be found in the book *Topics in Advanced Model Rocketry* by G. J. Caporaso, G. K. Mandell and W. P. Bengen, MIT Press, 1971.