A MATLAB Implementation of the IAU 2000B Nutation Theory

This document describes a MATLAB function and companion demonstration script that calculates the nutation in longitude and obliquity based on the IAU 2000B theory. This nutation theory is based on the numerical method described in "An Abridged Model of the Precession-Nutation of the Celestial Pole", by D. McCarthy and B. Luzum, *Celestial Mechanics and Dynamical Astronomy*, **85**: 37-49, 2003.

This function requires initialization the first time it is called. This can be accomplished by placing the following statement in the main script along with a `global inutate` statement.

```
inutate = 1;
```

The nutation in longitude is determined from a series of the form

$$\Delta\psi = \sum_{i=1}^{78} \left(A_i + A_i' t\right)\sin\theta + A_i''\cos\theta$$

Likewise, the nutation in obliquity is determined from

$$\Delta\varepsilon = \sum_{i=1}^{78} \left(B_i + B_i' t\right)\cos\theta + B_i''\sin\theta$$

where

$$\theta = \sum_{1}^{5} N_i F_i$$

In this last summation, N_i are integer multipliers and F_i are fundamental arguments. The fundamental arguments are given by the following expressions;

$$F_1 = l = \text{ mean anomaly of the Moon}$$
$$= 485868''.249036 + 1717915923''.2178t$$

$$F_2 = l' = \text{ mean anomaly of the Sun}$$
$$= 1287104''.79305 + 129596581''.0481t$$

$$F_3 = F = L - \Omega \;\; (L \text{ is the mean longitude of the Moon})$$
$$= 335779''.526232 + 1739527262''.8478t$$

$$F_4 = D = \text{ mean elongation of the Moon from the Sun}$$
$$= 1072260''.70369 + 1602961601''.2090t$$

$$F_5 = \Omega = \text{ mean longitude of the ascending node of the lunar orbit}$$
$$= 450160''.398036 - 6962890''.5431t$$

In these equations, the time argument *t* is the number of Julian centuries since J2000 and is given by $t = (JD - 2451545.0)/36525$ where *JD* is the Terrestrial Time (TT) Julian Date. Additional information about these arguments can be found in "Numerical Expressions for Precession Formulae and Mean Elements for the Moon and Planets", *Astronomy and Astrophysics*, **282**: 663-683.

The nutation matrix defined by

$$\mathbf{N} = \begin{bmatrix} \cos\Delta\psi & -\sin\Delta\psi\cos\varepsilon_0 & -\sin\Delta\psi\sin\varepsilon_0 \\ \sin\Delta\psi\cos\varepsilon & \cos\Delta\psi\cos\varepsilon\cos\varepsilon_0 + \sin\varepsilon\sin\varepsilon_0 & \cos\Delta\psi\cos\varepsilon\cos\varepsilon_0 - \sin\varepsilon\cos\varepsilon_0 \\ \sin\Delta\psi\sin\varepsilon & \cos\Delta\psi\sin\varepsilon\cos\varepsilon_0 - \cos\varepsilon\sin\varepsilon_0 & \cos\Delta\psi\sin\varepsilon\sin\varepsilon_0 + \cos\varepsilon\cos\varepsilon_0 \end{bmatrix}$$

can be used to transform a mean equinox of date position vector $\mathbf{r}_0$ to a true equinox of date position vector $\mathbf{r}$ as follows:

$$\mathbf{r} = [\mathbf{N}]\mathbf{r}_0$$

In this matrix ε_0 is the mean obliquity of the ecliptic and $\varepsilon = \varepsilon_0 + \Delta\varepsilon$ is the true obliquity.

The nutation matrix can also be expressed as a combination of individual rotations according to

$$\mathbf{N} = \mathbf{R}_1(-\varepsilon)\mathbf{R}_3(-\Delta\psi)\mathbf{R}_1(+\varepsilon_0)$$

where

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \qquad R_3(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The mean obliquity of the ecliptic is calculated from

$$\varepsilon_0 = 23^0 26' 21.'' 448 - 46''.8150T - 0''.00059T^2 + 0''.001813T^3$$

where $T = (JD - 2451545.0)/36525$ and *JD* is the Julian Date.

If second-order terms are neglected, a <u>linearized</u> nutation matrix can be calculated from

$$\mathbf{N} = \begin{bmatrix} 1 & -\Delta\psi\cos\varepsilon & -\Delta\psi\sin\varepsilon \\ \Delta\psi\cos\varepsilon & 1 & -\Delta\varepsilon \\ \Delta\psi\sin\varepsilon & \Delta\varepsilon & 1 \end{bmatrix}$$

Mean equinox equatorial rectangular position coordinates can be converted to true equinox coordinates by adding the following corrections to the respective components:

$$\Delta r_x = -\left(r_y \cos\varepsilon + r_z \sin\varepsilon\right)\Delta\psi$$

$$\Delta r_y = r_x \cos\varepsilon\,\Delta\psi - r_z \Delta\varepsilon$$

$$\Delta r_z = r_x \sin\varepsilon\,\Delta\psi + r_y \Delta\varepsilon$$

Finally, the nutation corrections to right ascension and declination are given by

$$\Delta\alpha = \left(\cos\varepsilon + \sin\varepsilon \sin\alpha \tan\delta\right)\Delta\psi - \cos\alpha \tan\delta\,\Delta\varepsilon$$

$$\Delta\delta = \sin\varepsilon \cos\alpha\,\Delta\psi + \sin\alpha\,\Delta\varepsilon$$

The syntax of this MATLAB function is

```
function [dpsi, deps] = nut2000b (jdate)

% nutation based on iau 2000b theory

% input

%  jdate = tt julian date

% output

%  dpsi = nutation in longitude in arc seconds

%  deps = nutation in obliquity in arc seconds

% reference

%  An Abridged Model of the Precession-Nutation
%  of the Celestial Pole, D. McCarthy and B. Luzum,
%  Celestial Mechanics and Dynamical Astronomy, 85: 37-49, 2003.

% Orbital Mechanics with Matlab

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

The `demo_nut2000b.zip` archive includes a script called `demo_nut2000b` that demonstrates how to interact with this MATLAB function. The following is a typical user interaction with this script.

```
demo_nut2000b - demonstrates how to use the nut2000b.m function

please input a UTC calendar date and time

please input the calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
? 12,28,2012
```

```
please input the universal time
(0 <= hours <= 24, 0 <= minutes <= 60, 0 <= seconds <= 60)
? 12,14,51
```

The following is the script output for this example.

```
program demo_nut2000b

UTC calendar date       28-Dec-2012

UTC Julian date          2456290.01031250

TT Julian date           2456290.01107852


nutation in obliquity        -6.05007782 arc seconds

nutation in longitude        14.45551185 arc seconds
```

The following are the results for this same calendar year and time using the Multiyear Interactive Computer Almanac (MICA) published by the United States Naval Observatory.

```
                          NUTATION AND OBLIQUITY

   Date         Time         Obliq. of Ecliptic          Nutation in
        (UT1)                      Mean     True         Long.      Obliq.
               h  m   s      °  '    "        "            "          "
2012 Dec 28 12:14:51.0    23 26  15.3214  9.2713    +14.4557   - 6.0501
```

An excellent online resource for these types of astronomical calculations can be found at the Data Services site of the U. S. Naval Observatory which is located at http://aa.usno.navy.mil/data/.

Time systems

Coordinated Universal Time, UTC

Coordinated Universal Time (UTC) is the time scale available from broadcast time signals. It is a compromise between the highly stable atomic time and the irregular earth rotation. UTC is the international basis of civil and scientific time.

Terrestrial Time, TT

Terrestrial Time is the time scale that would be kept by an ideal clock on the geoid - approximately, sea level on the surface of the Earth. Since its unit of time is the SI (atomic) second, TT is independent of the variable rotation of the Earth. TT is meant to be a smooth and continuous "coordinate" time scale independent of Earth rotation. In practice TT is derived from International Atomic Time (TAI), a time scale kept by real clocks on the Earth's surface, by the relation **TT = TAI + $32^s.184$.** It is the time scale now used for the precise calculation of future astronomical events observable from Earth.

TT = TAI + 32.184 seconds

TT = UTC + (number of leap seconds) + 32.184 seconds