

## Appendix C

### Time Scales

This appendix is a brief explanation of the time scales used in the `npoe` MATLAB script.

#### *Coordinated Universal Time, UTC*

Coordinated Universal Time (UTC) is the time scale available from broadcast time signals. It is a compromise between the highly stable atomic time and the irregular earth rotation. UTC is the international basis of civil and scientific time.

#### *Terrestrial Time, TT*

Terrestrial Time is the time scale that would be kept by an ideal clock on the geoid - approximately, sea level on the surface of the Earth. Since its unit of time is the SI (atomic) second, TT is independent of the variable rotation of the Earth. TT is meant to be a smooth and continuous “coordinate” time scale independent of Earth rotation. In practice TT is derived from International Atomic Time (TAI), a time scale kept by real clocks on the Earth's surface, by the relation **TT = TAI + 32<sup>s</sup>.184**. It is the time scale now used for the precise calculation of future astronomical events observable from Earth.

$$TT = TAI + 32.184 \text{ seconds}$$

$$TT = UTC + (\text{number of leap seconds}) + 32.184 \text{ seconds}$$

#### *Barycentric Dynamical Time, TDB*

Barycentric Dynamical Time is the time scale that would be kept by an ideal clock, free of gravitational fields, co-moving with the solar system barycenter. It is always within 2 milliseconds of TT, the difference caused by relativistic effects. TDB is the time scale now used for investigations of the dynamics of solar system bodies.

$$TDB = TT + \text{periodic corrections}$$

where typical periodic corrections (USNO Circular 179) are

$$\begin{aligned} TDB = TT &+ 0.001657 \sin(628.3076T + 6.2401) \\ &+ 0.000022 \sin(575.3385T + 4.2970) \\ &+ 0.000014 \sin(1256.6152T + 6.1969) \\ &+ 0.000005 \sin(606.9777T + 4.0212) \\ &+ 0.000005 \sin(52.9691T + 0.4444) \\ &+ 0.000002 \sin(21.3299T + 5.5431) \\ &+ 0.000010T \sin(628.3076T + 4.2490) + \dots \end{aligned}$$

In this equation, the coefficients are in seconds, the angular arguments are in radians, and  $T$  is the number of Julian centuries of  $TT$  from J2000;  $T = (\text{Julian Date}(TT) - 2451545.0) / 36525$ .

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The following is the MATLAB source code that performs these calculations.

```
function jdtddb = utc2tdb(jdutc)

% convert UTC julian day to TDB julian day

% input

% jdutc = UTC julian day

% output

% jdtddb = TDB julian day

% 'Reference Frames in Astronomy and Geophysics'
% J. Kovalevsky et al., 1989, pp. 439-442

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dtr = pi / 180.0;

deltat = find_deltat(jdutc);

% TDT julian day

corr = deltat / 86400.0;

jdtddt = jdutc + corr;

% time argument for correction

t = (jdtddt - 2451545.0) / 36525.0;

% compute correction in microseconds

corr = 1656.675      * sin(dtr * (35999.3729 * t + 357.5287)) ...
      + 22.418      * sin(dtr * (32964.467  * t + 246.199)) ...
      + 13.84       * sin(dtr * (71998.746  * t + 355.057)) ...
      +  4.77       * sin(dtr * ( 3034.906  * t +  25.463)) ...
      +  4.677      * sin(dtr * (34777.259  * t + 230.394)) ...
      + 10.216 * t * sin(dtr * (35999.373  * t + 243.451)) ...
      +  0.171 * t * sin(dtr * (71998.746  * t + 240.980)) ...
      +  0.027 * t * sin(dtr * ( 1222.114  * t + 194.661)) ...
      +  0.027 * t * sin(dtr * ( 3034.906  * t + 336.061)) ...
      +  0.026 * t * sin(dtr * ( -20.186   * t +   9.382)) ...
      +  0.007 * t * sin(dtr * (29929.562  * t + 264.911)) ...
      +  0.006 * t * sin(dtr * ( 150.678   * t +  59.775)) ...
      +  0.005 * t * sin(dtr * ( 9037.513  * t + 256.025)) ...
      +  0.043 * t * sin(dtr * (35999.373  * t + 151.121));

% convert corrections to days

corr = 0.000001 * corr / 86400.0;

% TDB julian day

jdtddb = jdtddt + corr;
```

## Leap seconds

The difference between International Atomic Time (TAI) and Universal Coordinated Time (UTC) is the number of current leap seconds. International Atomic Time (TAI, Temps Atomique International) is a physical time scale with the unit of the SI (System International) second and derived from a statistical timescale based on many atomic clocks. Coordinated Universal Time (UTC) is the time scale available from broadcast time signals. It is a compromise between the highly stable atomic time and the irregular earth rotation. UTC is the international basis of civil and scientific time.

The value of `deltat` in the code is read and interpolated from a simple text file with the following form

```
2433647.50, 29.5700
2434012.50, 29.9700
2434378.50, 30.3600
2434743.50, 30.7200
2435108.50, 31.0700
2435473.50, 31.3500
2435839.50, 31.6800
2436204.50, 32.1800
2436569.50, 32.6800
```

In this file column one is the Julian day and column two is the value of  $\Delta t$  in seconds. This file should be updated as leap seconds are added by time keeping services.

Here's the MATLAB source code for the routine that computes the value of  $\Delta t$ .

```
function deltat = find_deltat(jdutc)

% find delta-t corresponding to utc julian day

% input

% jdutc = utc julian day

% input via global

% jday_dt = array of utc julian days
% delta_t = array of leap seconds

% output

% deltat = delta-t (seconds)

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global jday_dt delta_t

% length of data arrays

ndata = length(jday_dt);

if (jdutc <= jday_dt(1))

    % day is <= beginning of current data

    deltat = delta_t(1);
```

```
elseif (jdutc >= jday_dt(ndata))  
    % day is >= end of current data  
    deltat = delta_t(ndata);  
else  
    % find data within table  
    for i = 1:1:ndata - 1  
        if (jdutc >= jday_dt(i) && jdutc < jday_dt(i + 1))  
            deltat = delta_t(i);  
            break;  
        end  
    end  
end
```