A MATLAB Script for Earth-to-Mars Mission Design

This document describes a MATLAB script named `e2m_matlab.m` that can be used to design and optimize ballistic interplanetary missions from Earth park orbit to encounter at Mars. The software assumes that interplanetary injection occurs *impulsively* from a circular Earth park orbit. The B-plane coordinates are expressed in a Mars-centered (areocentric) mean equator and IAU node of epoch coordinate system. B-plane targets are enforced using either a combination of periapsis radius and orbital inclination, individual B-plane coordinates ($\mathbf{B \bullet T}$ and $\mathbf{B \bullet R}$) of the arrival hyperbola, entry interface (EI) conditions at Mars or an areocentric node/apse aligned orbit. The type of targeting and the target values are defined by the user.

The first part of this MATLAB script solves for the minimum delta-v using a *patched-conic*, two-body Lambert solution for the transfer trajectory from Earth to Mars. Using this solution as an initial guess, the second part implements a simple *shooting* method that attempts to optimize the characteristics of the geocentric injection hyperbola while numerically integrating the spacecraft's geocentric and heliocentric equations of motion and targeting to components of the B-plane relative to Mars.

The spacecraft motion within the Earth's SOI includes the Earth's J_2 oblate gravity effect and the point-mass perturbations of the sun and moon. The heliocentric equations of motion include the point-mass gravity of the sun and the first seven planets of the solar system.

The user can select one of the following delta-v optimization options for the two-body solution of the interplanetary transfer trajectory:

- minimize departure delta-v
- minimize arrival delta-v
- minimize total delta-v
- no optimization

The major computational steps implemented in this script are as follows:

- solve the two-body, patched-conic interplanetary Lambert problem for the energy C_3, declination (DLA) and asymptote (RLA) of the outgoing or departure hyperbola
- compute the orbital elements of the geocentric departure hyperbola and the components of the interplanetary injection delta-v vector using the two-body solution
- perform geocentric orbit propagation from perigee of the geocentric departure hyperbola to the Earth's sphere-of-influence (SOI; default value = 925,000 kilometers)
- perform an n-body heliocentric orbit propagation from the Earth's SOI to closest approach at Mars
- target to the user-defined B-plane coordinates while minimizing the magnitude of the hyperbolic v-infinity at Earth departure (equivalent to minimizing the departure energy since $C_3 = V_\infty^2$)

This MATLAB script uses the SNOPT nonlinear programming algorithm to solve both the patched-conic and numerically integrated trajectory optimization problems. The coordinates of the sun, moon and planets are computed using the JPL Development Ephemeris DE421.

Input file format and contents

The `e2m_matlab` script is "data driven" by a simple text file created by the user. This section describes a typical simulation definition data file for this MATLAB script. In the following discussion the actual input file contents are in *`courier`* font and all explanations are in *times italic* font. Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation.

The annotation line also includes the correct units and when appropriate, the valid range of the input. The time scale for all internal calculations is Barycentric Dynamical Time (TDB).

The software allows the user to specify an initial guess for the departure and arrival calendar dates and a search interval. For any departure time guess t_L and user-defined search interval Δt, the departure time t is constrained as follows:

$$t_L - \Delta t \le t \le t_L + \Delta t$$

Likewise, for any guess for arrival time t_A and user-defined search interval, the arrival time t is constrained as follows:

$$t_A - \Delta t \le t \le t_A + \Delta t$$

For fixed departure and/or arrival times, the search interval should be set to zero.

The first six lines of an input file are reserved for user comments. These lines are ignored by the software. However, the input file must begin with six and only six initial text lines.

```
*****************************************
** interplanetary trajectory optimization
** script ==> e2m_matlab.m
** Mars '03 mars03.in
** March 21, 2020
*****************************************
```

The first numerical input is an integer that defines the type of patched-conic trajectory optimization performed by this script. Please note that option 4 simply solves Lambert's two-point boundary value problem (TPBVP) using the inputs for departure and arrival calendar dates provided by the user.

```
*******************
* simulation type *
*******************
 1 = minimize departure delta-v
 2 = minimize arrival delta-v
 3 = minimize total delta-v
 4 = no optimization
--------------------
1
```

The next input defines an initial guess for the departure calendar date. Please be sure to include all digits of the calendar year.

```
departure calendar date initial guess (month, day, year)
6,1,2003
```

This next number defines the lower and upper search interval for the departure calendar date.

```
departure date search boundary (days)
30
```

The next input defines an initial guess for the arrival calendar date. Please be sure to include all digits of the calendar year.

```
arrival calendar date initial guess (month, day, year)
12,1,2003
```

This number defines the lower and upper search interval for the arrival calendar date.

```
arrival date search boundary (days)
30
```

The next set of inputs defines several characteristics of the departure hyperbola and initial flight conditions. The perigee altitude and launch site latitude are with respect to a spherical Earth. The launch azimuth is measured positive clockwise from north.

```
***************************
* geocentric phase modeling
***************************

perigee altitude of launch hyperbola (kilometers)
185.32

launch azimuth (degrees)
93.0

launch site latitude (degrees)
28.5
```

The next input specifies the type of targeting at Mars performed by the `e2m_matlab` *script. Option 1 will target to components of the B-plane and option 2 will target to a Mars-centered hyperbola with a specified radius of closest approach (periapsis) and orbital inclination. Option 3 will target to user-defined entry interface (EI) conditions, option 4 will target a grazing flyby of Mars with a user-defined B-plane angle and option 5 will target to alignment of the node and line of apsides using an areocentric radius of closest approach defined by the user.*

```
****************************
* encounter planet targeting
****************************

type of targeting
(1 = B-plane, 2 = orbital elements, 3 = EI conditions, 4 = grazing flyby, 5 = node/apse
alignment)
2
```

The next two inputs are the user-defined B-plane components used with targeting option 1.

```
B dot T
4607.4

B dot R
-7888.0
```

These next two inputs define the radius of closest approach and the orbital inclination of the encounter hyperbola at Mars. These flight conditions are used by targeting option 2. The radius of closest approach is with respect to a spherical Mars model and the orbital inclination is with respect to the mean equator of Mars.

```
radius of closest approach (kilometers)
5000.0

orbital inclination (degrees)
60.0
```

The next two inputs define the inertial flight path angle and altitude of the entry interface (EI) at Mars. These flight conditions are used by targeting option 3. The altitude is with respect to a spherical Mars model. Targeting option 3 will also target to the orbital inclination defined above.

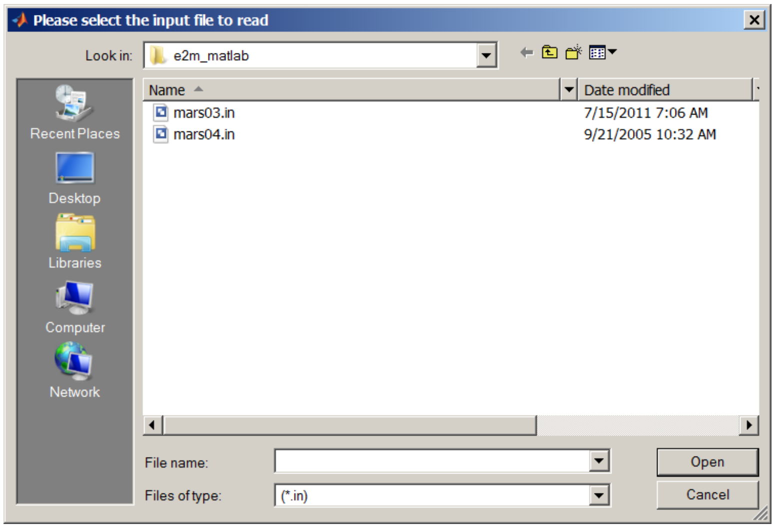
```
EI flight path angle (degrees)
-2.0

EI altitude (kilometers)
100.0
```

The final input is the user-defined B-plane angle for the grazing flyby option. Please note that the B-plane angle is measured positive clockwise from the **T** axis of the B-plane coordinate system.

```
user-defined b-plane angle (degrees)
-60.0
```

User interaction with the script

The `e2m_matlab` script will interactively prompt the user for the name of a simulation definition data file with a window like the following



Please select the input file to read

Look in: e2m_matlab

Name	Date modified
mars03.in	7/15/2011 7:06 AM
mars04.in	9/21/2005 10:32 AM

File name:

Files of type: (*.in)

The default filename type is `*.in`. However, the script will read any compatible data file.

Script example

The following is the solution created with this MATLAB script for this example. The output is organized by the following major sections

- Two-body/patched-conic pass
    1. two body Lambert solution
    2. departure hyperbola orbital elements and state vector
    3. heliocentric coordinates of Earth at departure and Mars at arrival
    4. heliocentric coordinates of the spacecraft on the transfer trajectory

- Targeting/optimization pass
    1. optimized characteristics of the departure hyperbola
    2. heliocentric coordinates of the spacecraft and Mars at closest approach
    3. geocentric and heliocentric coordinates of the spacecraft at the Earth SOI

The first output section illustrates the two-body Lambert solution. The solution is provided in the heliocentric, Earth mean equator and equinox of J2000 (EME2000) coordinate system. The time scale is Universal Coordinated Time (UTC). Please see Appendix A for additional explanation about the provided information.

```
Earth-to-Mars mission design

=========================
two-body Lambert solution
=========================

minimize departure delta-v

departure heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       2895.912029  meters/second
y-component of delta-v       -530.392173  meters/second
z-component of delta-v       -345.714443  meters/second

delta-v magnitude            2964.311187  meters/second

arrival heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v      -2063.021249  meters/second
y-component of delta-v       1164.269967  meters/second
z-component of delta-v       1311.950036  meters/second

delta-v magnitude            2707.913242  meters/second

heliocentric coordinates of the Earth at departure
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003
UTC time             14:45:51.038
UTC Julian Date      2452796.11517405

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +1.49651470774498e+08  +1.62373593524946e-02  +2.34390546573581e+01  +1.02452362635220e+02

       raan (deg)         true anomaly (deg)        arglat (deg)          period (days)
 +7.24187819311138e-04  +1.52047567978704e+02  +2.54499930613924e+02  +3.65453220144497e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -4.05623630692609e+07  -1.34199555373413e+08  -5.81817477377645e+07  +1.51789135744312e+08
```

```
        vx (kps)                vy (kps)                vz (kps)              vmag (kps)
 +2.82279377718406e+01  -7.39781819630137e+00  -3.20746513476093e+00  +2.93569756192143e+01

spacecraft heliocentric coordinates after the first impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003
UTC time             14:45:51.038
UTC Julian Date      2452796.11517405

        sma (km)             eccentricity       inclination (deg)         argper (deg)
 +1.88387151459821e+08  +1.94277214581077e-01  +2.34900370189654e+01  +2.53490942012329e+02

       raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 +4.55961022390640e-01  +5.91396440164278e-01  +2.54082338452493e+02  +5.16163425410496e+02

        rx (km)                 ry (km)                 rz (km)               rmag (km)
 -4.05623630692609e+07  -1.34199555373413e+08  -5.81817477377645e+07  +1.51789135744312e+08

        vx (kps)                vy (kps)                vz (kps)              vmag (kps)
 +3.11238498007479e+01  -7.92821036931141e+00  -3.55317957823716e+00  +3.23137065530213e+01

spacecraft heliocentric coordinates prior to the second impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003
UTC time             15:22:10.176
UTC Julian Date      2452998.14039555

        sma (km)             eccentricity       inclination (deg)         argper (deg)
 +1.88387151459821e+08  +1.94277214581077e-01  +2.34900370189654e+01  +2.53490942012329e+02

       raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 +4.55961022390640e-01  +1.52909961893317e+02  +4.64009039056457e+01  +5.16163425410496e+02

        rx (km)                 ry (km)                 rz (km)               rmag (km)
 +1.49990743136453e+08  +1.46776399958959e+08  +6.32690770191712e+07  +2.19188299684074e+08

        vx (kps)                vy (kps)                vz (kps)              vmag (kps)
 -1.46793471860696e+01  +1.56263777985786e+01  +6.84186616235204e+00  +2.25050671960032e+01

spacecraft heliocentric coordinates after the second impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003
UTC time             15:22:10.176
UTC Julian Date      2452998.14039555

        sma (km)             eccentricity       inclination (deg)         argper (deg)
 +1.88387151459821e+08  +1.94277214581077e-01  +2.34900370189654e+01  +2.53490942012329e+02

       raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 +4.55961022390640e-01  +1.52909961893317e+02  +4.64009039056457e+01  +5.16163425410496e+02

        rx (km)                 ry (km)                 rz (km)               rmag (km)
 +1.49990743136453e+08  +1.46776399958959e+08  +6.32690770191712e+07  +2.19188299684074e+08

        vx (kps)                vy (kps)                vz (kps)              vmag (kps)
 -1.67423684353423e+01  +1.67906477654654e+01  +8.15381619829816e+00  +2.50742392068893e+01

heliocentric coordinates of Mars at arrival
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003
UTC time             15:22:10.176
UTC Julian Date      2452998.14039555

        sma (km)             eccentricity       inclination (deg)         argper (deg)
 +2.27939307066468e+08  +9.35418899496124e-02  +2.46772249522146e+01  +3.32979237117963e+02
```

```
       raan (deg)         true anomaly (deg)        arglat (deg)          period (days)
 +3.37165832648902e+00  +7.07595401441841e+01  +4.37387772621471e+01  +6.86972171057758e+02

        rx (km)                ry (km)                rz (km)               rmag (km)
 +1.49990743136453e+08  +1.46776399958959e+08  +6.32690770191712e+07  +2.19188299684074e+08

        vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 -1.67423684353423e+01  +1.67906477654654e+01  +8.15381619829816e+00  +2.50742392068893e+01
```

The following output summarizes the orbital characteristics of the initial circular park orbit and the departure hyperbola for the two-body solution.

```
park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003
UTC time             14:45:51.038
UTC Julian Date      2452796.11517405

park orbit
----------

        sma (km)            eccentricity       inclination (deg)        argper (deg)
 +6.56345780000000e+03  +0.00000000000000e+00  +2.86442848562298e+01  +0.00000000000000e+00

       raan (deg)         true anomaly (deg)        arglat (deg)          period (hrs)
 +2.03558256806981e+00  +1.95039779371059e+02  +1.95039779371059e+02  +1.46996679905864e+00

        rx (km)                ry (km)                rz (km)               rmag (km)
 -6.28154038683986e+03  -1.71891122987610e+03  -8.16439124911574e+02  +6.56345780000000e+03

        vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 +2.25552459478573e+00  -6.52893504343690e+00  -3.60775100125299e+00  +7.79296165049872e+00

departure hyperbola
-------------------

c3                             8.787141  kilometer^2/second^2

v-infinity                  2964.311187  meters/second

asymptote right ascension     349.621193  degrees

asymptote declination          -6.697394  degrees

perigee altitude              185.320000  kilometers

launch azimuth                 93.000000  degrees

launch site latitude           28.500000  degrees

        sma (km)            eccentricity       inclination (deg)        argper (deg)
 -4.53617905974500e+04  +1.14469132971944e+00  +2.86442848562298e+01  +1.95039779371059e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.03558256806981e+00  +0.00000000000000e+00  +1.95039779371059e+02

        rx (km)                ry (km)                rz (km)               rmag (km)
 -6.28154038683986e+03  -1.71891122987610e+03  -8.16439124911574e+02  +6.56345780000000e+03

        vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 +3.30316269544242e+00  -9.56147174200788e+00  -5.28346644915040e+00  +1.14126089648719e+01

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1047.638101  meters/second
y-component of delta-v      -3032.536699  meters/second
z-component of delta-v      -1675.715448  meters/second

delta-v magnitude            3619.647314  meters/second
```

The following program output summarizes the flight conditions determined by the n-body, numerically integrated optimized solution. The initial screen illustrates the SNOPT algorithm characteristics.

```
 Nonzero derivs  Jij         7
 Non-constant    Jij's       6     Constant Jij's           1

 SNJAC  EXIT 100 -- finished successfully
 SNJAC  INFO 102 -- Jacobian structure estimated

 Scale option  0


 Nonlinear constraints       2     Linear constraints       1
 Nonlinear variables         3     Linear variables         0
 Jacobian  variables         3     Objective variables      0
 Total constraints           3     Total variables          3

 Itn      0: Feasible linear rows
 Itn      0: PP1.  Minimizing  Norm(x-x0)

 Itn      0: PP1.  Norm(x-x0) approximately minimized  (0.00E+00)


 The user has defined       0   out of       6   first  derivatives

 Itn      0: Hessian set to a scaled identity matrix

 Major Minors     Step   nCon Feasible  Optimal  MeritFunction     nS Penalty
     0      1              1  6.9E+00  1.3E-03  2.9643112E+00                  r
 Itn      1: Hessian set to a scaled identity matrix
     1      0  1.3E-01      2  6.0E+00  1.6E-04  2.9642649E+00                  r
     2      1  2.7E-02      3  5.8E+00  3.8E-03  2.9644116E+00      1 2.2E-07 s
     3      1  1.3E-01      4  5.1E+00  2.4E-03  2.9645811E+00      1 5.0E-07
     4      1  1.6E-01      5  4.3E+00  2.5E-03  2.9646274E+00      1 6.0E-07
     5      1  2.3E-01      6  3.3E+00  2.7E-03  2.9646563E+00      1 7.1E-07
     6      1  3.5E-01      7  2.1E+00  2.2E-03  2.9646792E+00      1 8.5E-07
     7      1  4.9E-01      8  1.1E+00  1.5E-03  2.9646915E+00      1 1.0E-06
     8      1  4.1E-01      9  6.4E-01  4.5E-04  2.9646949E+00      1 1.2E-06
     9      1  3.7E-01     10  4.0E-01  6.0E-04  2.9646950E+00      1 1.3E-06

 Major Minors     Step   nCon Feasible  Optimal  MeritFunction     nS Penalty
    10      1  1.0E+00     11  6.4E-04  9.0E-05  2.9646954E+00      1 1.5E-06
    11      1  1.0E+00     12  2.1E-05  1.4E-05  2.9646953E+00      1 1.5E-06
    12      1  1.0E+00     13 (9.7E-08)(1.1E-06) 2.9646953E+00      1 1.5E-06

 SNOPTA EXIT   0 -- finished successfully
 SNOPTA INFO   1 -- optimality conditions satisfied

 Problem name                 matlabMx
 No. of iterations                  12   Objective            2.9646953458E+00
 No. of major iterations            12   Linear    obj. term  2.9646953458E+00
 Penalty parameter           1.503E-06   Nonlinear obj. term  0.0000000000E+00
 User function calls (total)        78   Calls with modes 1,2 (known g)     13
 Calls for forward differencing     39   Calls for central differencing      0
 No. of superbasics                  1   No. of basic nonlinears             2
 No. of degenerate steps             0   Percentage                       0.00
 Max x                       2 6.1E+00   Max pi                      3 1.0E+00
 Max Primal infeas           0 0.0E+00   Max Dual infeas             2 6.7E-06
 Nonlinear constraint violn    5.9E-07

=======================
optimal n-body solution
=======================

radius of closest approach and inclination

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003
UTC time             14:45:51.038
UTC Julian Date      2452796.11591693
```

```
park orbit
----------

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +6.56345780000000e+03  +0.00000000000000e+00  +2.86442848562298e+01  +0.00000000000000e+00

       raan (deg)         true anomaly (deg)        arglat (deg)          period (hrs)
 +2.67612391703511e+00  +1.94740980977338e+02  +1.94740980977338e+02  +1.46996679905864e+00

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27207315457118e+03  -1.76043889495244e+03  -8.00581996566851e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +2.28956784023919e+00  -6.51430084250126e+00  -3.61275724682320e+00  +7.79296165049872e+00

departure hyperbola
-------------------

c3                              8.789418  km^2/sec^2

v-infinity                   2964.695346  meters/second

asymptote right ascension     349.992812  degrees

asymptote declination          -6.838784  degrees


        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -4.53500355907832e+04  +1.14472883459730e+00  +2.86442848562298e+01  +1.94740980977338e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.67612391703511e+00  +0.00000000000000e+00  +1.94740980977338e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27207315457118e+03  -1.76043889495244e+03  -8.00581996566853e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.35304754485751e+00  -9.54012371353455e+00  -5.29084423869317e+00  +1.14127087523397e+01

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1063.479705  meters/second
y-component of delta-v      -3025.822871  meters/second
z-component of delta-v      -1678.086992  meters/second

delta-v magnitude            3619.747102  meters/second

transfer time                 201.331199  days

spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003
UTC time             18:20:36.728
UTC Julian Date      2452799.26431399

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -4.56025217203415e+04  +1.14327373027494e+00  +2.85069555886698e+01  +1.94699397014455e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.68905716456695e+00  +1.49478601381027e+02  +3.44177998395482e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +8.99372529847290e+05  -1.79627063194135e+05  -1.20366817372295e+05  +9.25000000000000e+05

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.03072155836363e+00  -5.32311681783782e-01  -3.66011314417739e-01  +3.09880512023189e+00

spacecraft heliocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------
```

```
UTC calendar date    08-Jun-2003
UTC time             18:20:36.728
UTC Julian Date      2452799.26431399

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +1.90703472044679e+08  +2.03962410923551e-01  +2.35000171967760e+01  +2.53589673071939e+02

       raan (deg)         true anomaly (deg)        arglat (deg)          period (days)
 +5.26390465608453e-01  +3.79011732527619e+00  +2.57379790397215e+02  +5.25712385594130e+02

        rx (km)                 ry (km)                 rz (km)               rmag (km)
 -3.19302901241574e+07  -1.36202193341066e+08  -5.90923972842418e+07  +1.51863400182956e+08

        vx (kps)                vy (kps)                vz (kps)              vmag (kps)
 +3.16290640102060e+01  -6.53281191338549e+00  -2.96677776907924e+00  +3.24326547169436e+01

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             22:42:46.653
UTC Julian Date      2452997.44637330

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -5.84497984089590e+03  +1.85543482595794e+00  +6.00000115334248e+01  +1.13895638355991e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +1.05732420585564e+02  +8.28667851362374e-06  +1.13895646642670e+02

        rx (km)                 ry (km)                 rz (km)               rmag (km)
 -1.65091791208966e+03  -2.56924658498966e+03  +3.95896895192495e+03  +4.99999931292448e+03

        vx (kps)                vy (kps)                vz (kps)              vmag (kps)
 +2.19016114762413e+00  -4.08068269901870e+00  -1.73492178883112e+00  +4.94557688808836e+00

b-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude                 9135.085304  kilometers
b dot r                    -7888.070909  kilometers
b dot t                     4607.398490  kilometers
b-plane angle                300.289072  degrees
v-infinity                  2706.912987  meters/second
r-periapsis                 4999.999313  kilometers
decl-asymptote                 7.541851  degrees
rasc-asymptote               281.348533  degrees

flight path angle              0.000005  degrees

spacecraft heliocentric coordinates at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             22:42:46.653
UTC Julian Date      2452997.44637330

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +1.84408725531071e+08  +2.37019433485322e-01  +1.90415886597410e+01  +2.71353427542515e+02

       raan (deg)         true anomaly (deg)        arglat (deg)          period (hrs)
 +3.43910704359844e+02  +1.50099736221541e+02  +6.14531637640565e+01  +1.19975826412996e+04
        rx (km)                 ry (km)                 rz (km)               rmag (km)
 +1.50993436081283e+08  +1.45762045017182e+08  +6.27803213748497e+07  +2.19059261979842e+08

        vx (kps)                vy (kps)                vz (kps)              vmag (kps)
 -1.35220426984716e+01  +1.70345561281454e+01  +4.35562733640814e+00  +2.21809204208457e+01

heliocentric coordinates of Mars at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             22:42:46.653
UTC Julian Date      2452997.44637330

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +2.27939359811927e+08  +9.35421331918644e-02  +2.46772248803922e+01  +3.32979308926179e+02

       raan (deg)         true anomaly (deg)        arglat (deg)          period (days)
 +3.37165816301200e+00  +7.03676488082284e+01  +4.33469577344079e+01  +6.86972409507183e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +1.50991266569169e+08  +1.45766253590466e+08  +6.27787147880226e+07  +2.19060106583553e+08

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 -1.66285413267696e+01  +1.69012860960416e+01  +8.20148694472973e+00  +2.50883687485039e+01
```

The final program output summarizes the verification of the optimal solution. Appendix E discusses how this numerical verification is performed.

```
====================================
verification of the optimal solution
====================================

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date         05-Jun-2003
UTC time                  14:45:51.038
UTC Julian Date           2452796.11591693

park orbit
----------

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +6.56345780000000e+03  +0.00000000000000e+00  +2.86442848562298e+01  +0.00000000000000e+00

       raan (deg)         true anomaly (deg)        arglat (deg)          period (hrs)
 +2.67612391703511e+00  +1.94740980977338e+02  +1.94740980977338e+02  +1.46996679905864e+00

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27207315457118e+03  -1.76043889495244e+03  -8.00581996566851e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +2.28956784023919e+00  -6.51430084250126e+00  -3.61275724682320e+00  +7.79296165049872e+00

departure hyperbola
-------------------

c3                              8.789418  kilometers^2/second^2

v-infinity                   2964.695346  meters/second

asymptote right ascension     349.992812  degrees

asymptote declination          -6.838784  degrees

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -4.53500355907832e+04  +1.14472883459730e+00  +2.86442848562298e+01  +1.94740980977338e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.67612391703511e+00  +0.00000000000000e+00  +1.94740980977338e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27207315457118e+03  -1.76043889495244e+03  -8.00581996566853e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.35304754485751e+00  -9.54012371353455e+00  -5.29084423869317e+00  +1.14127087523397e+01

spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date         08-Jun-2003
UTC time                  18:20:36.728
UTC Julian Date           2452799.26431399

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -4.56025217169717e+04  +1.14327373026335e+00  +2.85069555886794e+01  +1.94699397013259e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.68905716456125e+00  +1.49478601382173e+02  +3.44177998395432e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +8.99372529799998e+05  -1.79627063185499e+05  -1.20366817366405e+05  +9.24999999951575e+05

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.03072155847099e+00  -5.32311681809966e-01  -3.66011314434666e-01  +3.09880512034339e+00

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

UTC calendar date         23-Dec-2003
UTC time                  22:42:46.653
UTC Julian Date           2452997.44637330

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -5.84498024594246e+03  +1.85543474434185e+00  +6.00000257076313e+01  +1.13895639725033e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +1.05732418363297e+02  +7.42999176171718e-05  +1.13895714024950e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -1.65091442186080e+03  -2.56925104018060e+03  +3.95896735121172e+03  +4.99999918237368e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +2.19016127142701e+00  -4.08068180918405e+00  -1.73492370807993e+00  +4.94557688197466e+00

b-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude                 9135.085370  kilometers
b dot r                    -7888.072112  kilometers
b dot t                     4607.396563  kilometers
b-plane angle                300.289058  degrees
v-infinity                  2706.912893  meters/second
r-periapsis                 4999.999182  kilometers
decl-asymptote                 7.541853  degrees
rasc-asymptote               281.348532  degrees

flight path angle              0.000048  degrees
```

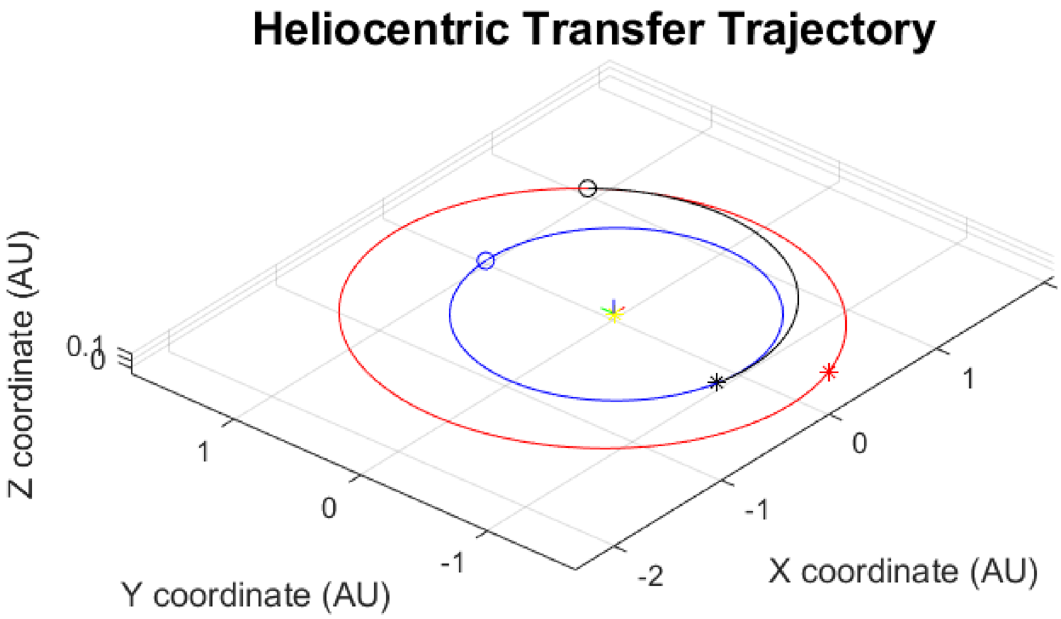
This MATLAB script will create a screen display and TIFF disk file of the interplanetary and encounter trajectories using a function named `marsplot.m`. The interactive graphic features of MATLAB allow the user to rotate and "zoom" the displays. These capabilities allow the user to interactively find the "best" viewpoint as well as verify orbital geometry of the heliocentric and areocentric trajectories.

The script will ask you for the step size at which to plot the graphics data with the following prompt

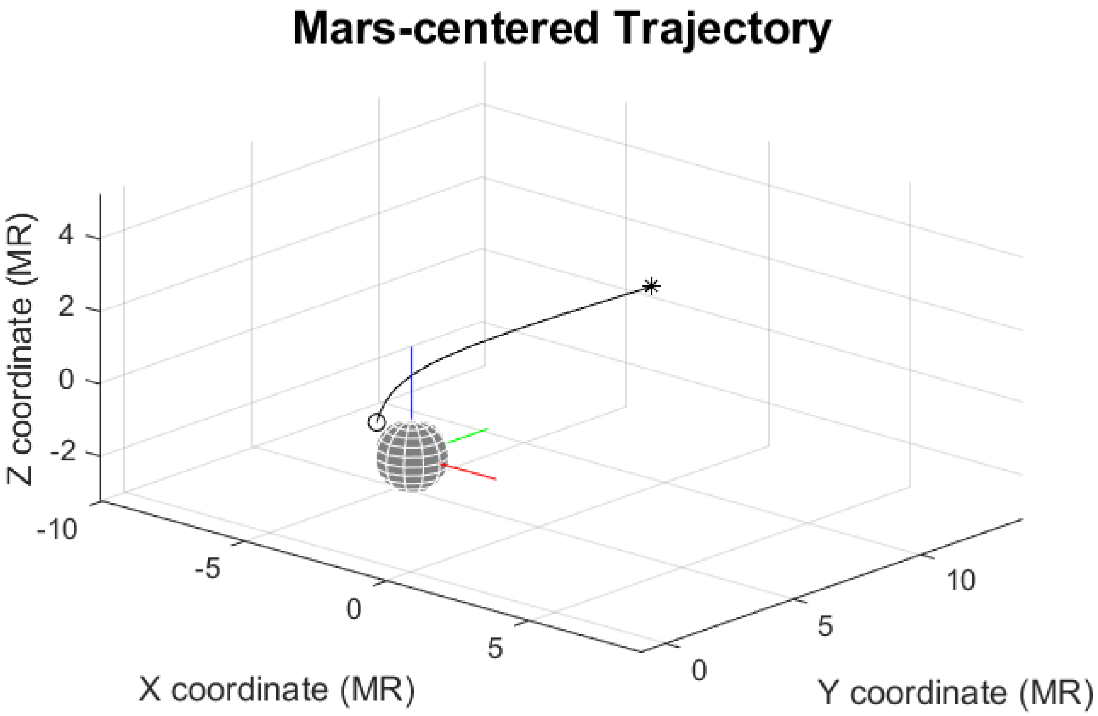
```
please input the plot step size (days)
?
```

A value between 1 and 5 days is recommended. A "hard-wired" value of 5 minutes is used with the spacecraft is within 50,000 kilometers of Mars.

The following is a heliocentric, above the ecliptic plane view of the transfer and planetary orbits. The x-axis of this system is red, the y-axis green and the z-axis is blue. The orbit of the Earth is blue, the orbit of Mars is red and the transfer trajectory is black. The beginning of each orbit and the transfer trajectory is marked with an asterisk. The locations of the spacecraft and planets at arrival are marked with a small circle. The scale of this display is astronomical units (AU).



The next plot is a view of the encounter trajectory in the Mars-centered mean equator and IAU node of epoch coordinate system. The small black dot is the periapsis of the encounter hyperbola. The asterisk indicates the location of the spacecraft when it is 50,000 kilometers from Mars. This image is saved to disk with the filename `marsplot1.tif`. The scale of this display is the radius of Mars (MR).



This image is saved to disk with the filename `marsplot2.tif`. Please note the `marsplot` function can be run stand-alone from within the MATLAB command window. This allows the user to experiment with different graphic step sizes without having to re-run the main script.

Technical discussion

This section describes the main numerical methods implemented in the `e2m_matlab` script.

Solving the two body Lambert problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamics problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis *a* of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length *c* of the chord joining these two positions. This relationship can be stated as follows

$$tof = tof\left(r_i + r_f, c, a\right)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}}\left(E - e\sin E\right)$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}}\left[E - E_0 - e\left(\sin E - \sin E_0\right)\right]$$

where *E* is the eccentric anomaly associated with radius *r*, E_0 is the eccentric anomaly at r_0, and $t = 0$ when $r = r_0$.

At this point we introduce the following trigonometric sun and difference identities

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha - \beta}{2}\cos\frac{a + \beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha - \beta}{2}\sin\frac{a + \beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha - \beta}{2}\cos\frac{a + \beta}{2}$$

If we let $E = \alpha$, $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following time-of-flight equation

$$t = \sqrt{\frac{a^3}{\mu}}\left\{E - E_0 - 2\sin\frac{E - E_0}{2}\left(e\cos\frac{E + E_0}{2}\right)\right\}$$

With the two substitutions given by

$$e\cos\frac{E+E_0}{2}=\cos\frac{\alpha+\beta}{2}$$

$$\sin\frac{E-E_0}{2}=\sin\frac{\alpha-\beta}{2}$$

the time equation becomes

$$t=\sqrt{\frac{a^3}{\mu}\left\{(\alpha-\beta)-2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}\right\}}$$

From the elliptic relationships given by

$$r=a(1-e\cos E)$$

$$x=a(\cos E-e)$$

$$y=a\sin E\sqrt{1-e^2}$$

and some more manipulation, we have the following equations

$$\cos\alpha=\left(1-\frac{r+r_0}{2a}\right)-\frac{c}{2a}=1-\frac{r+r_0+c}{2a}=1-\frac{s}{a}$$

$$\sin\beta=\left(1-\frac{r+r_0}{2a}\right)+\frac{c}{2a}=1-\frac{r+r_0-c}{2a}=1-\frac{s-c}{a}$$

This part of the derivation makes use of the following three relationships

$$\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}=1-\frac{r+r_0}{2}$$

$$\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}=\sin\frac{E-E_0}{2}\sqrt{1-\left(e\cos\frac{E+E_0}{2}\right)^2}$$

$$\left(\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\right)^2=\left(\frac{x-x_0}{2a}\right)^2+\left(\frac{y-y_0}{2a}\right)^2=\left(\frac{c}{2a}\right)^2$$

With the use of the half angle formulas given by

$$\sin\frac{\alpha}{2}=\sqrt{\frac{s}{2a}}\qquad\sin\frac{\beta}{2}=\sqrt{\frac{s-c}{2a}}$$

and several additional substitutions, we have the following time-of-flight form of Lambert’s theorem

$$t = \sqrt{\frac{a^3}{\mu}}\left[(\alpha - \beta) - (\sin\alpha - \sin\beta)\right]$$

A discussion about the angles α and β can be found in “Geometrical Interpretation of the Angles α and β in Lambert’s Problem” by John E. Prussing, AIAA *Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this MATLAB script is based on the method described in “A Procedure for the Solution of Lambert’s Orbital Boundary-Value Problem” by Robert H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde and involve one or more revolutions about the central body.

Designing the departure hyperbola

This section describes the algorithm used to determine the *Earth-centered-inertial* (ECI) state vector of a departure hyperbola for interplanetary missions. In the discussion that follows, interplanetary injection is assumed to occur *impulsively* at perigee of the departure hyperbola.

The departure trajectory for interplanetary missions can be defined using the specific (per unit mass) orbital energy C_3, and the right ascension α_∞ (RLA) and declination δ_∞ (DLA) of the outgoing asymptote. The perigee radius of the departure hyperbola is calculated from the user’s value for the altitude of the circular park orbit and the radius of the Earth.

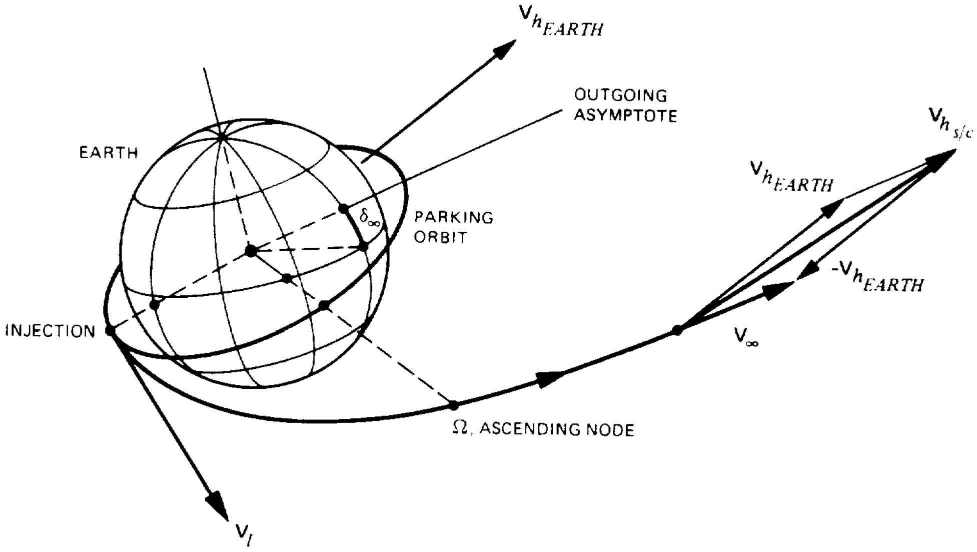
The following figure illustrates the geometry of interplanetary injection. In this diagram $\mathbf{V}_{h_{EARTH}}$ is the heliocentric velocity vector of the Earth at departure, $\mathbf{V}_{h_{s/c}}$ is the heliocentric velocity vector of the spacecraft at departure, $\mathbf{V}_\infty$ is the geocentric v-infinity or *excess velocity* vector and $\mathbf{V}_l$ is the velocity vector of the departure hyperbola at perigee.

From the velocity vector diagram, $\mathbf{V}_\infty = \mathbf{V}_{h_{s/c}} - \mathbf{V}_{h_{EARTH}}$. The heliocentric velocity vector of the spacecraft at departure is determined from the two-body Lambert solution for the heliocentric transfer trajectory. The heliocentric velocity vector of the Earth at departure is determined from a JPL ephemeris.

The two-body Lambert solution is used to initialize the n-body integrated solution by assuming the orientation of the outgoing asymptote of the departure hyperbola can be determined from the components of the geocentric $\mathbf{V}_\infty$ velocity vector.

The twice specific (per unit mass) orbital energy of the departure hyperbola is $C_3 = V_\infty^2$. The right ascension α_∞ and declination δ_∞ of the departure asymptote can be determined from the components of the v-infinity vector according to $\alpha_\infty = \tan^{-1}\left(V_{\infty_y}, V_{\infty_x}\right)$ and $\delta_\infty = \sin^{-1}\left(V_{\infty_z} / V_\infty\right)$. The inverse tangent used in the first equation is a four quadrant calculation.

Since the right ascension of the outgoing asymptote is an *inertial* coordinate, it should <u>not</u> be called the longitude of the asymptote. This terminology also applies to the right ascension of the ascending node.



This figure was extracted from page 2 of *Interplanetary Mission Design Handbook, Volume 1, Part 2,* JPL Publication 82-43, September 15, 1983.

The orbital inclination is computed from the user-defined launch azimuth Σ_L (measured positive clockwise from north) and launch site geocentric latitude ϕ_L using this equation

$$i = \cos^{-1}\left(\cos\phi_L \sin\Sigma_L\right)$$

The algorithm used to design the departure hyperbola is valid for geocentric orbit inclinations that satisfy the following constraint

$$|i| > |\delta_\infty|$$

If this inequality is not satisfied, the software will print the following error message

```
park orbit error!!

|inclination| must be > |asymptote declination|
```

The code will also print the inclination of the park orbit, the declination of the departure hyperbola and pause. The user can then change either the azimuth, launch site latitude or orbital inclination to satisfy this constraint and restart the script.

A unit vector in the direction of the departure asymptote is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where

$\alpha_\infty =$ right ascension of departure asymptote (RLA)

$\delta_\infty =$ declination of departure asymptote (DLA)

The T-axis direction of the B-plane coordinate system is determined from the following vector cross product:

$$\hat{\mathbf{T}} = \hat{\mathbf{S}} \times \hat{\mathbf{u}}_z$$

where $\hat{\mathbf{u}}_z = [0\,0\,1]^T$ is a unit vector perpendicular to the Earth's equatorial plane.

The following cross product operation completes the B-plane coordinate system.

$$\hat{\mathbf{R}} = \hat{\mathbf{S}} \times \hat{\mathbf{T}}$$

The B-plane angle is determined from the orbital inclination of the departure hyperbola *i* and the declination of the outgoing asymptote according to

$$\cos\theta = \cos i / \cos\delta_\infty$$

The unit angular momentum vector of the departure hyperbola is given by

$$\hat{\mathbf{h}} = \hat{\mathbf{T}}\sin\theta - \hat{\mathbf{R}}\cos\theta$$

The sine and cosine of the true anomaly at infinity are given by the next two equations

$$\cos\theta_\infty = -\frac{\mu}{r_p V_\infty^2 + \mu} \qquad \sin\theta_\infty = \sqrt{1 - \cos^2\theta_\infty}$$

where $V_\infty = \sqrt{C_3} = V_L - V_p$ is the spacecraft's velocity at infinity (v-infinity), V_L is the heliocentric departure velocity determined from the Lambert solution, V_p is the heliocentric velocity of the departure planet, and r_p is the user-specified perigee radius of the departure hyperbola.

A unit vector in the direction of perigee of the departure hyperbola is determined from

$$\hat{\mathbf{r}}_p = \hat{\mathbf{S}}\cos\theta_\infty - \left(\hat{\mathbf{h}} \times \hat{\mathbf{S}}\right)\sin\theta_\infty$$

The ECI position vector at perigee is equal to $\mathbf{r}_p = r_p\,\hat{\mathbf{r}}_p$.

The scalar magnitude of the departure hyperbola perigee velocity can be determined from

$$V_p = \sqrt{\frac{2\mu}{r_p} + V_\infty^2} = \sqrt{V_{lc}^2 + V_\infty^2}$$

where $V_{lc} = \sqrt{\mu / r_p}$ is the local circular velocity.

A unit vector aligned with the velocity vector at perigee is

$$\hat{\mathbf{v}}_p = \hat{\mathbf{h}} \times \hat{\mathbf{r}}_p$$

The ECI velocity vector at perigee of the departure hyperbola is given by

$$\mathbf{v}_p = V_p \hat{\mathbf{v}}_p$$

Finally, the classical orbital elements of the departure hyperbola can be determined from the position and velocity vectors at perigee. The impulsive injection delta-v vector and magnitude can be determined from the velocity difference between the local circular velocity of the park orbit and the geocentric velocity of the departure hyperbola each evaluated at the orbital location of the propulsive maneuver.

Propagating the spacecraft's trajectory

The spacecraft's orbital motion is modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The *z*-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time (TT).

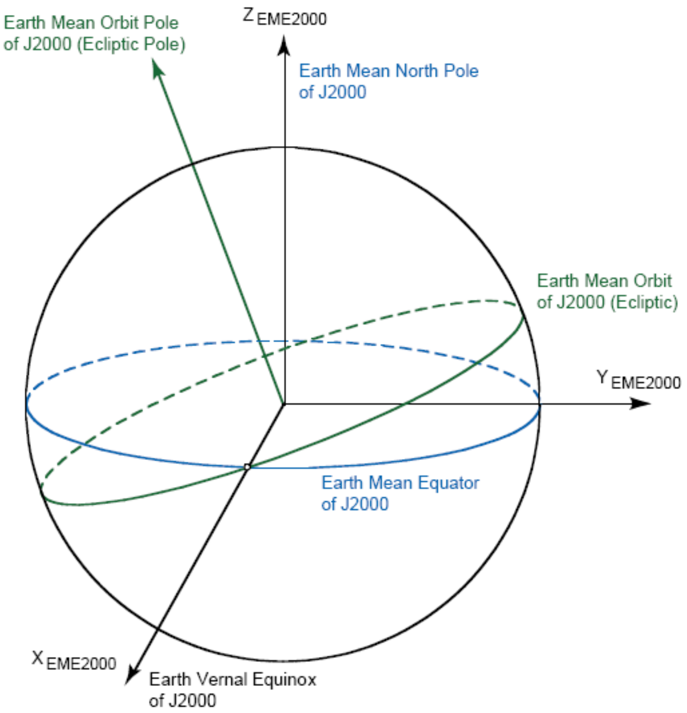


Figure 1. Earth mean equator and equinox of J2000 coordinate system

Geocentric trajectory propagation

This part of the trajectory analysis implements a *special perturbation* technique which numerically integrates the vector system of second-order, nonlinear differential equations of motion of a spacecraft given by

$$\vec{a}\left(\vec{r},\vec{v},t\right) = \ddot{\vec{r}}\left(\vec{r},\dot{\vec{r}},t\right) = \vec{a}_g\left(\vec{r}\right) + \vec{a}_m\left(\vec{r},t\right) + \vec{a}_s\left(\vec{r},t\right)$$

where

$t =$ time

$\vec{r} =$ inertial position vector of the satellite

$\vec{v} =$ inertial velocity vector of the satellite

$\vec{a}_g =$ acceleration due to Earth gravity

$\vec{a}_m =$ acceleration due to the Moon

$\vec{a}_s =$ acceleration due to the Sun

The system of six first-order differential equations subject to Earth gravity is defined by

$$\dot{y}_1 = v_x = y_4 \qquad \dot{y}_2 = v_y = y_5 \qquad \dot{y}_2 = v_z = y_6$$

$$\dot{y}_4 = -\mu \frac{r_x}{r^3}\left\{1 + \frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(1 - \frac{5r_z^2}{r^2}\right)\right\}$$

$$\dot{y}_5 = -\mu \frac{r_y}{r^3}\left\{1 + \frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(1 - \frac{5r_z^2}{r^2}\right)\right\}$$

$$\dot{y}_6 = -\mu \frac{r_z}{r^3}\left\{1 + \frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(3 - \frac{5r_z^2}{r^2}\right)\right\}$$

where $r = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{y_1^2 + y_2^2 + y_3^2}$. In these equations μ and r_{eq} are the gravitational constant and equatorial radius of the Earth, respectively and J_2 is the oblateness gravity coefficient.

The acceleration contribution of the Moon represented by a *point mass* is given by

$$\vec{a}_m\left(\vec{r}, t\right) = -\mu_m \left(\frac{\vec{r}_{m-b}}{\left|\vec{r}_{m-b}\right|^3} + \frac{\vec{r}_{e-m}}{\left|\vec{r}_{e-m}\right|^3} \right)$$

where

$\mu_m =$ gravitational constant of the Moon

$\vec{r}_{m-b} =$ position vector from the Moon to the satellite

$\vec{r}_{e-m} =$ position vector from the Earth to the Moon

The acceleration contribution of the sun represented by a *point mass* is given by

$$\mathbf{a}_s\left(\mathbf{r}, t\right) = -\mu_s \left(\frac{\mathbf{r}_{s-sc}}{\left|\mathbf{r}_{s-sc}\right|^3} + \frac{\mathbf{r}_{e-s}}{\left|\mathbf{r}_{e-s}\right|^3} \right)$$

where

$\mu_s =$ gravitational constant of the sun

$\mathbf{r}_{s-sc} =$ position vector from the sun to the spacecraft

$\mathbf{r}_{e-s} =$ position vector from the Earth to the sun

The `e2m_matlab` script uses Professor Battin's $f(q)$ function described in the next section to compute the point-mass gravity of the Sun and Moon.

Heliocentric trajectory propagation

The general vector equation for *point-mass* perturbations such as the Moon or planets is given by

$$\ddot{\mathbf{r}} = -\sum_{j=1}^{n} \mu_j \left[\frac{\mathbf{d}_j}{d_j^3} + \frac{\mathbf{s}_j}{s_j^3} \right]$$

In this equation, $\mathbf{s}_j$ is the vector from the primary body to the secondary body *j*, μ_j is the gravitational constant of the secondary body and $\mathbf{d}_j = \mathbf{r} - \mathbf{s}_j$, where $\mathbf{r}$ is the position vector of the spacecraft relative to the primary body.

To avoid numerical problems, use is made of Professor Battin's $f(q)$ function given by

$$f(q_k) = q_k \left[\frac{3 + 3q_k + q_k^2}{1 + \left(\sqrt{1 + q_k}\right)^3} \right]$$

where

$$q_k = \frac{\mathbf{r}^T (\mathbf{r} - 2\mathbf{s}_k)}{\mathbf{s}_k^T \mathbf{s}_k}$$

The point-mass acceleration due to *n* gravitational bodies can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^{n} \frac{\mu_k}{d_k^3} \left[\mathbf{r} + f(q_k)\, \mathbf{s}_k \right]$$

In these equations, $\mathbf{s}_k$ is the vector from the primary body to the secondary body, μ_k is the gravitational constant of the secondary body and $\mathbf{d}_k = \mathbf{r} - \mathbf{s}_k$, where $\mathbf{r}$ is the position vector of the spacecraft relative to the primary body. The derivation of the $f(q)$ functions is described in Section 8.4 of "An *Introduction to the Mathematics and Methods of Astrodynamics*, Revised Edition", by Richard H. Battin, AIAA Education Series, 1999.

In this MATLAB script the heliocentric coordinates of the Moon, planets and sun are based on the JPL Development Ephemeris DE421. These coordinates are evaluated in the Earth mean equator and equinox of J2000 coordinate system (EME2000). A binary ephemeris file can be downloaded from http://celestialandorbitalmechanicswebsite.yolasite.com/.

Predicting the conditions at the Earth's sphere of influence

The trajectory conditions at the boundary of the Earth's sphere of influence are determined during the numerical integration of the spacecraft's geocentric equations of motion by finding the time at which the difference between the geocentric distance and the user-defined SOI value is essentially zero. This scalar mission constraint is computed as follows

$$\Delta r = \left|\mathbf{r}_{sc}\right|_p - r_{soi_u} \approx 0$$

where $\left|\mathbf{r}_{sc}\right|_p$ is the scalar magnitude of the *predicted* geocentric position vector of the spacecraft and r_{soi_u} is the user-defined value of the geocentric distance of the SOI boundary.

In this script, the sphere-of-influence calculations use the *event-finding* feature of the `ode45` MATLAB function. The following is the source code that performs this calculation.

```
%-----------------------------------------------------
% solve for geocentric sphere-of-influence conditions
%-----------------------------------------------------

% set up for ode45

options = odeset('RelTol', 1.0e-10, 'AbsTol', 1.0e-10, 'Events', @soi_event);

% define maximum search time (seconds)

tof = 25.0 * 86400.0;

[t, ysol, tevent, yevent, ie] = ode45(@e2m_eqm1, [0 tof], [rhyper vhyper], options);
```

The following is the MATLAB source code that evaluates the SOI event function.

```
function [value, isterminal, direction] = soi_event(t, y)

% sphere-of-influence event function

% input

%  t = current simulation time
%  y = current spacecraft geocentric state vector (km & km/sec)

% output

%  value = difference between current position and soi (kilometers)

% global

%  rsoi = Earth sphere-of-influence (kilometers)

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

global rsoi

% difference between current geocentric distance and soi (kilometers)

value = norm(y(1:3)) - rsoi;

isterminal = 1;

direction =  [];
```
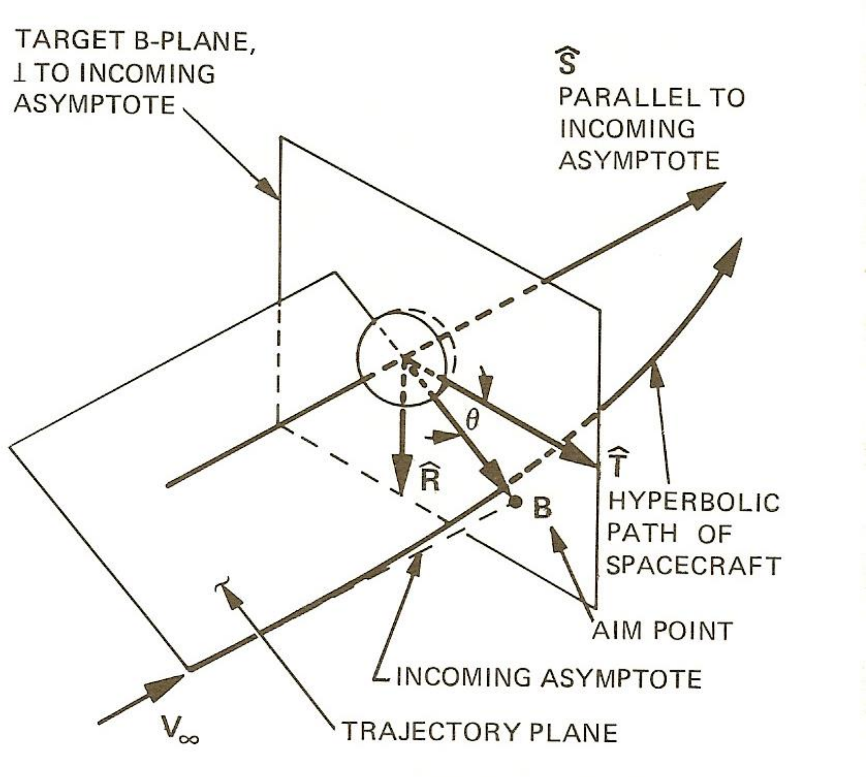

This script uses a “hard-wired” value of 925,000 kilometers for distance of the Earth’s SOI.

B-plane targeting

The derivation of B-plane coordinates is described in the classic JPL reports, “A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories” and “Some Orbital Elements Useful in Space Trajectory Calculations”, both written by William Kizner. The following diagram illustrates the fundamental geometry of the B-plane coordinate system as it relates to the incoming trajectory.



Given the user-defined closest approach radius r_{ca} and orbital inclination i, the incoming v-infinity magnitude v_∞, and the right ascension α_∞ (RLA) and declination δ_∞ (DLA) of the incoming asymptote vector at moment of closest approach, the following series of equations can be used to determine the required B-plane *target* vector

$$\mathbf{B{\bullet}T} = b_t \cos\theta \qquad \mathbf{B{\bullet}R} = b_t \sin\theta$$

where

$$b_t = \sqrt{\frac{2\mu\, r_{ca}}{v_\infty^2} + r_{ca}^2} = r_{ca}\sqrt{1 + \frac{2\mu}{r_{ca} v_\infty^2}}$$

and

$$\cos\theta = \frac{\cos i}{\cos\delta_\infty} \qquad \sin\theta = -\sqrt{1-\cos^2\theta} \qquad \to \theta = \tan^{-1}\left(\sin\theta, \cos\theta\right)$$

The cosine of the B-plane angle can also be determined from

$$\cos\theta = \frac{\cos i}{\sqrt{\hat{S}_x^2 + \hat{S}_y^2}}$$

Furthermore

$$\sin\delta_\infty = \left|\hat{\mathbf{s}} \times \hat{\mathbf{z}}\right| = \sqrt{s_x^2 + s_y^2}$$

and

$$\hat{\mathbf{z}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola.

Important note

This technique is valid for areocentric orbit inclinations that satisfy $|i| > |\delta_\infty|$. If this inequality is not satisfied, the software will print the following error message and pause

```
b-plane targeting error!!

|inclination| must be > |asymptote declination|
```

It will also display the actual declination of the asymptote. The user should then edit the input file, include a different orbital inclination and restart the simulation.

As noted in the thesis of R. H. See, the minimum and maximum values of orbital inclination can be determined from the *x* and *y* components of the unit $\hat{\mathbf{S}}$ vector according to

$$i_{\min} = \cos^{-1}\left(\sqrt{\hat{S}_x^2 + \hat{S}_y^2}\right) \qquad i_{\max} = \cos^{-1}\left(-\sqrt{\hat{S}_x^2 + \hat{S}_y^2}\right)$$

For an arrival hyperbola that lies in the equatorial plane of Mars, the following condition must be true

$$i_{\min} = 0 \rightarrow \sqrt{\hat{S}_x^2 + \hat{S}_y^2} = 1$$

Furthermore, if the *z* component of the incoming v-infinity vector is positive, $\mathbf{V}_{\infty_z} > 0$, an equatorial areocentric orbit is not possible.

B-plane equations

The following computational steps summarize the calculation of the *predicted* B-plane vector from an inertial planet-centered position vector **r** and velocity vector **v** at closest approach.

angular momentum vectors

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} \qquad \hat{\mathbf{h}} = \frac{\mathbf{h}}{|\mathbf{h}|}$$

radius rate

$$\dot{r} = \mathbf{r} \bullet \mathbf{v} \big/ |\mathbf{r}|$$

semi-parameter

$$p = h^2 \big/ \mu$$

semimajor axis

$$a = \frac{r}{\left(2 - \dfrac{r\,v^2}{\mu}\right)}$$

orbital eccentricity

$$e = \sqrt{1 - p/a}$$

true anomaly

$$\cos\theta = \frac{p - r}{e\,r} \qquad \sin\theta = \frac{\dot{r}\,h}{e\mu} \quad \rightarrow \theta = \tan^{-1}\left(\sin\theta, \cos\theta\right)$$

B-plane magnitude

$$B = \sqrt{p|a|}$$

fundamental vectors

$$\hat{\mathbf{z}} = \frac{r\,\mathbf{v} - \dot{r}\,\mathbf{r}}{h}$$

$$\hat{\mathbf{p}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\mathbf{z}} \qquad \hat{\mathbf{q}} = \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\mathbf{z}}$$

S vector

$$\mathbf{S} = -\frac{a}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{b}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

B vector

$$\mathbf{B} = \frac{b^2}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{ab}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

T vector

$$\mathbf{T} = \frac{\left(S_y^2, -S_x^2, 0\right)^T}{\sqrt{S_x^2 + S_y^2}}$$

R vector

$$\mathbf{R} = \mathbf{S} \times \mathbf{T} = \left(-S_z T_y,\ S_z T_x,\ S_x T_y - S_y T_x\right)^T$$

Targeting to the Mars-centered periapsis radius and orbital inclination

For this targeting option, the two equality mission constraints enforced by the nonlinear programming algorithm are

$$r_p - r_{ca} = 0$$

$$\cos i - \hat{\mathbf{h}}_z = 0$$

where r_p and i are the user-defined periapsis radius and orbital inclination, respectively. $\hat{\mathbf{h}}_z$ is the z-component of the *predicted* unit angular momentum vector and r_{ca} is the predicted distance, both evaluated at closest approach to Mars.

The mission elapsed time at which the spacecraft reaches closest approach to Mars is predicted using the event prediction capability of the MATLAB `ode45` algorithm. During the numerical integration of the spacecraft's heliocentric equations of motion, the `ode45` numerical method searches for the time at which the flight path angle *with respect to Mars* is zero within a small tolerance.

Closest approach is predicted with the flight path angle mission constraint given by $\sin\gamma = \mathbf{r}\bullet\mathbf{v} / |\mathbf{r}\bullet\mathbf{v}|$ where **r** and **v** are the Mars-centered-inertial position and velocity vectors, respectively.

Targeting to user-defined B-plane coordinates

For this targeting option, the two nonlinear equality constraints enforced by the SNOPT nonlinear programming algorithm are

$$(\mathbf{B}\bullet\mathbf{T})_p - (\mathbf{B}\bullet\mathbf{T})_u = 0$$

$$(\mathbf{B}\bullet\mathbf{R})_p - (\mathbf{B}\bullet\mathbf{R})_u = 0$$

where the *p* subscript refers to coordinates predicted by the software and the *u* subscript denotes coordinates provided by the user. The *predicted* B-plane coordinates are based on the Mars-centered flight conditions at closest approach.

Targeting to user-defined entry interface (EI) conditions with user-defined inclination

For this targeting option, the following equations can be used to determine the required B-plane components based on the user-defined EI targets. These targets are the inertial flight path angle and altitude relative to a spherical Mars model.

$$\mathbf{B}\bullet\mathbf{T} = b_t \cos\theta \qquad \mathbf{B}\bullet\mathbf{R} = b_t \sin\theta$$

where

$$b_t = \cos\gamma_{ei}\sqrt{\frac{2\mu\, r_{ei}}{v_\infty^2} + r_{ei}^2} \quad \text{and} \quad \cos\theta = \frac{\cos i}{\cos\delta_\infty}$$

Note that this targeting option could be modified to use a user-defined B-plane angle instead of orbital inclination at the entry interface.

In these equations, γ_{ei} is the user-defined flight path angle at the entry interface, r_{ei} is the Mars-centered radius at the entry interface (sum of Mars equatorial radius plus user-defined EI altitude) and *i* is the user-defined areocentric orbital inclination.

Entry interface at Mars is determined during the numerical integration of the spacecraft's heliocentric equations of motion by finding the time at which the difference between the *predicted* Mars-centered flight path angle and the user-defined *inertial* entry interface flight path angle is zero. This mission constraint is enforced as follows

$$\sin^{-1}\left(\frac{\mathbf{r}\bullet\mathbf{v}}{\left|\mathbf{r}\bullet\mathbf{v}\right|}\right)-\gamma_t=0$$

where **r** and **v** are the Mars-centered position and velocity vectors, respectively, and γ_t is the user-defined EI flight path angle *target*. The event prediction capability of the MATLAB `ode45` algorithm is used to determine the time at which the spacecraft reaches the entry interface condition at Mars.

Targeting to a Mars-centered grazing flyby with user-defined B-plane angle

The general expression for the periapsis radius of an encounter hyperbola at Mars is given by

$$\tilde{r}_p=\frac{1}{\tilde{v}_\infty^2}\left(\sqrt{1+\tilde{b}_\infty^2\tilde{v}_\infty^4}-1\right)$$

where the *normalized* quantities are

$$\begin{aligned}
\tilde{r}_p &= \text{ normalized periapsis radius} = r_p/r_m \\
\tilde{b}_\infty &= \text{ normalized b-plane magnitude} = b_\infty/r_m \\
\tilde{v}_\infty &= \text{ normalized v-infinity speed} = v_\infty/v_{lc} \\
v_{lc} &= \text{ local circular speed at Mars} = \sqrt{\mu_m/r_m} \\
r_m &= \text{radius of Mars} \\
\mu_m &= \text{gravitational constant of Mars}
\end{aligned}$$

For a grazing flyby, $\tilde{r}_p=1$ and the normalized B-plane distance or offset is equal to

$$\tilde{b}_\infty=\sqrt{1+\frac{2}{\tilde{v}_\infty^2}}$$

Therefore, the required B-plane equality constraints are computed from

$$\mathbf{B}\bullet\mathbf{T}=b_\infty\cos\theta \qquad \mathbf{B}\bullet\mathbf{R}=b_\infty\sin\theta$$

where θ is the user-defined B-plane angle of the grazing flyby trajectory. Please note that the B-plane angle is measured positive clockwise from the **T** axis of the B-plane coordinate system. The two equality constraints for this program option are simply the difference between the predicted and required $\mathbf{B}\bullet\mathbf{T}$ and $\mathbf{B}\bullet\mathbf{R}$ components.

Targeting to a Mars-centered node/apse alignment trajectory

This targeting option determines when a spacecraft is simultaneously at a nodal crossing and periapsis of the Mars encounter hyperbola. The B-plane angle required for node-apse alignment is given by

$$\theta = \sin^{-1}\left(\frac{\sin\phi_h \hat{S}_z}{\cos\phi_h \sqrt{\hat{S}_x^2 + \hat{S}_y^2}}\right)$$

where $\phi_h = \sin^{-1}\left(1/e\right)$. This relationship is also developed in the thesis of R. H. See.

For all targeting options, the B-plane constraints are normalized by the radius of Mars.

EME2000-to-areocentric coordinate transformation

This section describes the transformation of coordinates between the Earth mean equator and equinox of J2000 and areocentric mean equator and IAU node of epoch coordinate systems. This transformation is used to compute the B-plane coordinates at encounter.

A unit vector in the direction of the pole of Mars can be determined from

$$\hat{\mathbf{p}}_{Mars} = \begin{bmatrix} \cos\alpha_p \cos\delta_p \\ \sin\alpha_p \cos\delta_p \\ \sin\delta_p \end{bmatrix}$$

The IAU 2000 right ascension and declination of the pole of Mars in the Earth mean equator and equinox of J2000 (EME2000) coordinate system are given by the following expressions

$$\alpha_p = 317.68143 - 0.1061T$$

$$\delta_p = 52.88650 - 0.0609T$$

where T is the time in Julian centuries given by $T = \left(JD - 2451545.0\right)/36525$ and JD is the TDB Julian day. The unit vector in the direction of the *IAU-defined* x-axis is computed from

$$\hat{\mathbf{x}} = \hat{\mathbf{p}}_{J2000} \times \hat{\mathbf{p}}_{Mars}$$

where $\hat{\mathbf{p}}_{J2000} = \left[0\,0\,1\right]^T$ is unit vector in the direction of the pole of the J2000 coordinate system.

The unit vector in the y-axis direction of this coordinate system is

$$\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Mars} \times \hat{\mathbf{x}}$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the Mars-centered mean equator and IAU node of epoch system are as follows

$$\mathbf{M} = \left[\hat{\mathbf{x}} \quad \hat{\mathbf{y}} \quad \hat{\mathbf{p}}_{Mars}\right]^T$$

SNOPT algorithm implementation

This section provides details about the parts of the MATLAB script that solve both nonlinear programming (NLP) problems using the SNOPT algorithm. In the classic two-body patched-conic trajectory optimization problem, the departure and arrival calendar dates are the *control variables* and the user-specified ΔV is the *objective function* or *performance index*.

MATLAB versions of SNOPT for several computer platforms can be requested or purchased at Professor Philip Gill's web site which is located at http://scicomp.ucsd.edu/~peg/. Professor Gill's web site also includes a PDF version of the SNOPT software user's guide.

The SNOPT algorithm requires an initial guess for the control variables. For the two-body Lambert problem these are given by

```
xg(1) = jdtdb_tip - jdtdb0;

xg(2) = jdtdb_arrival - jdtdb0;
```

where `jdtdb_tip` and `jdtdb_arrival` are the initial user-provided departure and arrival Julian day guesses, and `jdtdb0` is a reference Julian day (on the Barycentric Dynamical Time scale) equal to `2451544.5` (January 1, 2000). This offset value is used to *scale* the control variables.

The algorithm also requires lower and upper bounds for the control variables. These are determined from the initial guesses and user-defined search boundaries as follows:

```
% bounds on control variables

xlwr(1) = xg(1) - ddays1;

xupr(1) = xg(1) + ddays1;

xlwr(2) = xg(2) - ddays2;

xupr(2) = xg(2) + ddays2;
```

where `ddays1` and `ddays2` are the user-defined departure and arrival search boundaries, respectively.

Finally, the algorithm requires lower and upper bounds on the objective function. For this problem these bounds are given by

```
% bounds on objective function

flow(1) = 0.0;

fupp(1) = +Inf;
```

The actual call to the SNOPT MATLAB interface function (`snopt.m`) is as follows

```
[x, ~, ~, ~, ~] = snopt(xg, xlwr, xupr, xmul1, xstate1, ...
                        flow, fupp, fmul1, fstate1, 'e2m_deltav');
```

In the parameter list `e2m_deltav` is the name of the MATLAB function that solves Lambert's problem and computes the current value of the two-body objective function.

The following is the MATLAB source code snippet for the SNOPT initialization of the n-body optimization algorithm. The control variables for this part of the computations are the v-infinity magnitude, and the right ascension (RLA) and declination (DLA) of the outgoing asymptote. The objective function for these calculations is the scalar magnitude of the departure v-infinity. The initial guess vector `xg` uses the values of v-infinity, RLA and DLA computed by the two-body patched-conic Lambert solution.

The bounds for v-infinity are in units of kilometers per second and the bounds for RLA and DLA are in units of radians. The user can edit these bounds at this point in the source code.

```
% initial guess for launch vinf, rla and dla

xg = zeros(3, 1);

xg(1) = norm(twobody_dv1);

xg(2) = twobody_rasc1;

xg(3) = twobody_decl1;

% define lower and upper bounds for vinf, rla and dla

xlwr = zeros(3, 1);
xupr = zeros(3, 1);

xlwr(1) = xg(1) - 0.05;

xupr(1) = xg(1) + 0.05;

xlwr(2) = xg(2) - 10.0 * dtr;

xupr(2) = xg(2) + 10.0 * dtr;

xlwr(3) = xg(3) - 1.0 * dtr;

xupr(3) = xg(3) + 1.0 * dtr;

% bounds on objective function

flow(1) = 0.0;

fupp(1) = +Inf;

% bounds on final b-plane/orbital element equality constraints

flow(2) = 0.0;

fupp(2) = 0.0;

flow(3) = 0.0;

fupp(3) = 0.0;
```

The actual call to the SNOPT MATLAB interface function for this part of the script is as follows

```
[x, ~, inform, xmul, fmul] = snopt(xg, xlwr, xupr, xmul, xstate, ...
                                   flow, fupp, fmul, fstate, 'e2m_shoot');
```

where `e2m_shoot` is the name of the MATLAB function that implements an n-body simple shooting method that computes the time and flight characteristics at closest approach to Mars.

Time scales

This section is a brief explanation of the time scales used in this MATLAB script.

Coordinated Universal Time, UTC

Coordinated Universal Time (UTC) is the time scale available from broadcast time signals. It is a compromise between the highly stable atomic time and the irregular earth rotation. UTC is the international basis of civil and scientific time.

Terrestrial Time, TT

Terrestrial Time is the time scale that would be kept by an ideal clock on the geoid - approximately, sea level on the surface of the Earth. Since its unit of time is the SI (atomic) second, TT is independent of the variable rotation of the Earth. TT is meant to be a smooth and continuous "coordinate" time scale independent of Earth rotation. In practice TT is derived from International Atomic Time (TAI), a time scale kept by real clocks on the Earth's surface, by the relation **TT = TAI + 32^{s}.184.** It is the time scale now used for the precise calculation of future astronomical events observable from Earth.

TT = TAI + 32.184 seconds

TT = UTC + (number of leap seconds) + 32.184 seconds

Barycentric Dynamical Time, TDB

Barycentric Dynamical Time is the time scale that would be kept by an ideal clock, free of gravitational fields, co-moving with the solar system barycenter. It is always within 2 milliseconds of TT, the difference caused by relativistic effects. TDB is the time scale now used for investigations of the dynamics of solar system bodies.

TDB = TT + periodic corrections

where typical periodic corrections (USNO Circular 179) are

$$\begin{aligned} TDB = TT &+ 0.001657\sin\left(628.3076T + 6.2401\right) \\ &+ 0.000022\sin\left(575.3385T + 4.2970\right) \\ &+ 0.000014\sin\left(1256.6152T + 6.1969\right) \\ &+ 0.000005\sin\left(606.9777T + 4.0212\right) \\ &+ 0.000005\sin\left(52.9691T + 0.4444\right) \\ &+ 0.000002\sin\left(21.3299T + 5.5431\right) \\ &+ 0.000010T\sin\left(628.3076T + 4.2490\right) + \cdots \end{aligned}$$

In this equation the coefficients are in seconds, the angular arguments are in radians, and T is the number of Julian centuries of TT from J2000; T = (Julian day(TT) – 2451545.0) / 36525.

The following is the MATLAB source code for a function that computes a Julian day on the TDB scale from a Julian day on the UTC time scale.

```
function jdtdb = utc2tdb(jdutc, tai_utc)

% convert UTC julian day to TDB julian day

% input

%  jdutc   = UTC julian day
%  tai_utc = TAI-UTC (seconds)

% output

%  jdtdb = TDB julian day

% Reference Frames in Astronomy and Geophysics
% J. Kovalevsky et al., 1989, pp. 439-442

% Orbital Mechanics with MATLAB

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

dtr = pi / 180.0;

% TDT julian day

corr = (tai_utc + 32.184) / 86400.0;

jdtdt = jdutc + corr;

% time argument for correction

t = (jdtdt - 2451545.0) / 36525.0;

% compute correction in microseconds

corr = 1656.675     * sin(dtr * (35999.3729 * t + 357.5287))...
       + 22.418     * sin(dtr * (32964.467  * t + 246.199))...
       + 13.84      * sin(dtr * (71998.746  * t + 355.057))...
       +  4.77      * sin(dtr * ( 3034.906  * t +  25.463))...
       +  4.677     * sin(dtr * (34777.259  * t + 230.394))...
       + 10.216 * t * sin(dtr * (35999.373  * t + 243.451))...
       +  0.171 * t * sin(dtr * (71998.746  * t + 240.98 ))...
       +  0.027 * t * sin(dtr * ( 1222.114  * t + 194.661))...
       +  0.027 * t * sin(dtr * ( 3034.906  * t + 336.061))...
       +  0.026 * t * sin(dtr * (  -20.186  * t +   9.382))...
       +  0.007 * t * sin(dtr * (29929.562  * t + 264.911))...
       +  0.006 * t * sin(dtr * (  150.678  * t +  59.775))...
       +  0.005 * t * sin(dtr * ( 9037.513  * t + 256.025))...
       +  0.043 * t * sin(dtr * (35999.373  * t + 151.121));

% convert corrections to days

corr = 0.000001 * corr / 86400.0;

% TDB julian day

jdtdb = jdtdt + corr;
```

This algorithm can be found in “Reference Frames in Astronomy and Geophysics”, Jean Kovalevsky, Ivan I. Mueller, and Barbara Kolaczek, 1989, pp. 439-442, Springer Science & Business Media.

Leap seconds

The difference between International Atomic Time (TAI) and Universal Coordinated Time (UTC) is the number of current leap seconds. International Atomic Time (TAI**,** Temps Atomique International) is a physical time scale with the unit of the SI (System International) second and derived from a statistical timescale based on many atomic clocks. Coordinated Universal Time (UTC) is the time scale available from broadcast time signals. It is a compromise between the highly stable atomic time and the irregular earth rotation. UTC is the international basis of civil and scientific time.

The calculation of leap seconds in this MATLAB script is performed by a function that reads a simple text data file and evaluates the current value of leap seconds. The leap second function must be initialized by including the following statements in the main script.

```
% read leap seconds data file

readleap;
```

The `readleap` MATLAB function reads the contents of the following simple comma-separated-variable (csv) two column data file. The name of this file is `tai-utc.dat`.

```
2441317.5,  10.0
2441499.5,  11.0
2441683.5,  12.0
2442048.5,  13.0
2442413.5,  14.0
2442778.5,  15.0
2443144.5,  16.0
2443509.5,  17.0
2443874.5,  18.0
2444239.5,  19.0
2444786.5,  20.0
2445151.5,  21.0
2445516.5,  22.0
2446247.5,  23.0
2447161.5,  24.0
2447892.5,  25.0
2448257.5,  26.0
2448804.5,  27.0
2449169.5,  28.0
2449534.5,  29.0
2450083.5,  30.0
2450630.5,  31.0
2451179.5,  32.0
2453736.5,  33.0
2454832.5,  34.0
2456109.5,  35.0
2457204.5,  36.0
2457754.5,  37.0
```

The first column of this data file is the Julian day, on the UTC time scale, at which the leap second became valid. The second column is the leap second value, in seconds.

Note that this data is passed between the leap second MATLAB functions by way of a global statement.

```
global jdateleap leapsec
```

The source code for the MATLAB function that reads and evaluates the current value of leap seconds has the following syntax and single argument.

```
function leapsecond = findleap(jdate)

% find number of leap seconds for utc julian date

% input

%  jdate = utc julian date

% input via global
%  jdateleap = array of utc julian dates
%  leapsec   = array of leap seconds

% output

%  leapsecond = number of leap seconds

% Orbital Mechanics with MATLAB

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

global jdateleap leapsec

ndata = length(jdateleap);

if (jdate <= jdateleap(1))

    % date is <= 1972

    leapsecond = leapsec(1);

elseif (jdate >= jdateleap(ndata))

    % date is >= end of current data

    leapsecond = leapsec(ndata);

else

    % find data within table

   for i = 1:1:ndata - 1

       if (jdate >= jdateleap(i) && jdate < jdateleap(i + 1))

           leapsecond = leapsec(i);

           break;

       end

   end

end
```

Please note this function does <u>not</u> extrapolate outside the range of dates contained in the data file.

The leap seconds data file should be updated whenever the International Earth Rotation and Reference Systems Service (IERS) announces a new leap second.

Algorithm resources

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“A Procedure for the Solution of Lambert’s Orbital Boundary-Value Problem”, Robert H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990.

“A Complete and Fast Survey of the Orbital Insertion Design Space For Planetary Moon Missions”, Robert Harding See, MS Thesis, University of Texas, August 2017.

“A MATLAB Script for Ballistic Interplanetary Trajectory Design and Optimization”, https://www.mathworks.com/matlabcentral/fileexchange/73513-ballistic-interplanetary-trajectory-optimization-otb

“Parametric Analysis of Ballistic Earth-to-Mars Trajectories”, https://www.mathworks.com/matlabcentral/fileexchange/74254-parametric-analysis-of-ballistic-earth-to-mars-trajectories

“A MATLAB Script for Creating Pork Chop Plots of Ballistic Earth-to-Mars Trajectories”, https://www.mathworks.com/matlabcentral/fileexchange/authors/my_fileexchange

“Impulsive Hyperbolic Injection from an Earth Orbit”, https://www.mathworks.com/matlabcentral/fileexchange/73588-impulsive-hyperbolic-injection-from-an-earth-orbit-otb

“Finite-burn Interplanetary Injection from Earth Orbit”, https://www.mathworks.com/matlabcentral/fileexchange/73635-finite-burn-interplanetary-injection-from-earth-orbit-otb

APPENDIX A

Contents of the Simulation Summary

This appendix is a brief summary of the information contained in the simulation summary screen displays produced by the `e2m_matlab` software.

The simulation summary screen display contains the following information:

UTC calendar date = UTC calendar date of trajectory event

UTC time = UTC time of trajectory event

UTC Julian Date = Julian Date of trajectory event on UTC time scale

sma (km) = semimajor axis in kilometers

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of periapsis in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (days) = orbital period in days

rx (km) = x-component of the spacecraft’s position vector in kilometers

ry (km) = y-component of the spacecraft’s position vector in kilometers

rz (km) = z-component of the spacecraft’s position vector in kilometers

rmag (km) = scalar magnitude of the spacecraft’s position vector in kilometers

vx (kps) = x-component of the spacecraft’s velocity vector in kilometers per second

vy (kps) = y-component of the spacecraft’s velocity vector in kilometers per second

vz (ksp) = z-component of the spacecraft’s velocity vector in kilometers per second

vmag (kps) = scalar magnitude of the spacecraft’s velocity vector in kilometers per second

b-magnitude = magnitude of the b-plane vector in kilometers

b dot r = dot product of the b-vector and r-vector in kilometers

b dot t = dot product of the b-vector and t-vector in kilometers

b-plane angle = orientation of the b-plane vector in degrees

v-infinity = magnitude of outgoing (Earth departure) or incoming (Mars arrival) v-infinity vector in kilometers/second

r-periapsis = periapsis radius of incoming hyperbola in kilometers

decl-asymptote = declination of incoming v-infinity vector in degrees

rasc-asymptote = right ascension of incoming v-infinity vector in degrees

flight path angle = areocentric flight path angle in degrees

APPENDIX B

Entry Interface Example

This appendix summarizes the n-body trajectory characteristics of a typical entry interface simulation.

```
=======================
optimal n-body solution
=======================

entry interface with user-defined inclination

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003
UTC time             14:45:51.038
UTC Julian Date      2452796.11591693

park orbit
----------

        sma (km)             eccentricity       inclination (deg)         argper (deg)
 +6.56345780000000e+03  +0.00000000000000e+00  +2.86442848562298e+01  +0.00000000000000e+00

       raan (deg)         true anomaly (deg)        arglat (deg)         period (hrs)
 +2.71731435792592e+00  +1.94703070038279e+02  +1.94703070038279e+02  +1.46996679905864e+00

        rx (km)                 ry (km)                 rz (km)                rmag (km)
 -6.27208310184027e+03  -1.76131774247490e+03  -7.98568510544769e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)                vz (kps)               vmag (kps)
 +2.28932347288920e+00  -6.51403831855963e+00  -3.61338540802929e+00  +7.79296165049872e+00

departure hyperbola
-------------------

c3                             8.788735  km^2/sec^2

v-infinity                  2964.580044  meters/second

asymptote right ascension     350.001152  degrees

asymptote declination          -6.856039  degrees

        sma (km)             eccentricity       inclination (deg)         argper (deg)
 -4.53535632817281e+04  +1.14471757729881e+00  +2.86442848562298e+01  +1.94703070038279e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.71731435792592e+00  +0.00000000000000e+00  +1.94703070038279e+02

        rx (km)                 ry (km)                 rz (km)                rmag (km)
 -6.27208310184027e+03  -1.76131774247490e+03  -7.98568510544770e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)                vz (kps)               vmag (kps)
 +3.35268087264834e+00  -9.53971421381144e+00  -5.29175028626144e+00  +1.14126788006658e+01

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1063.357400  meters/second
y-component of delta-v      -3025.675895  meters/second
z-component of delta-v      -1678.364878  meters/second

delta-v magnitude            3619.717150  meters/second

transfer time                 201.353497  days
```

```
spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003
UTC time             18:20:45.317
UTC Julian Date      2452799.26441339

        sma (km)              eccentricity        inclination (deg)         argper (deg)
 -4.56061564903086e+04  +1.14326319194736e+00  +2.85079534435955e+01  +1.94661956058445e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.72982807521559e+00  +1.49479495393678e+02  +3.44141451452123e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +8.99362910061464e+05  -1.79490842546402e+05  -1.20641590870471e+05  +9.25000000000000e+05

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.03058087339523e+00  -5.31830900201433e-01  -3.66922343629604e-01  +3.09869271513854e+00

spacecraft heliocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003
UTC time             18:20:45.317
UTC Julian Date      2452799.26441339

        sma (km)              eccentricity        inclination (deg)         argper (deg)
 +1.90700752364615e+08  +2.03951053479988e-01  +2.35005260917957e+01  +2.53586028233554e+02

       raan (deg)         true anomaly (deg)        arglat (deg)         period (days)
 +5.30767374409945e-01  +3.78983898691496e+00  +2.57375867220469e+02  +5.25701139618521e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -3.19300541182041e+07  -1.36202108657377e+08  -5.90926943951432e+07  +1.51863390221558e+08

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.16289337101080e+01  -6.53228685522971e+00  -2.96766957595760e+00  +3.24325034789095e+01

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             23:14:53.191
UTC Julian Date      2452997.46867119

        sma (km)              eccentricity        inclination (deg)         argper (deg)
 -5.84771235899313e+03  +1.59728510865822e+00  +1.99997758508418e+01  +1.06180477070358e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +1.22671534759197e+02  +3.56747376525702e+02  +1.02927853596059e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -2.27321586636438e+03  -2.38699143222436e+03  +1.16545361675779e+03  +3.49621518293956e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +4.09203120753383e+00  -3.85123938576899e+00  -4.97032358431470e-01  +5.64125920121182e+00

b-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude                 7283.441871  kilometers
b dot r                    -2320.201941  kilometers
b dot t                     6903.998004  kilometers
b-plane angle                341.424232  degrees
v-infinity                  2706.280470  meters/second
r-periapsis                 3492.751512  kilometers
decl-asymptote                 7.546083  degrees
rasc-asymptote               281.327630  degrees

flight path angle             -2.000402  degrees
```

```
spacecraft heliocentric coordinates at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             23:14:53.191
UTC Julian Date      2452997.46867119

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +1.85445903590067e+08  +3.00857948623370e-01  +1.98345550547827e+01  +2.77767312077882e+02

       raan (deg)         true anomaly (deg)        arglat (deg)         period (hrs)
 +3.47955109551986e+02  +1.39884544539063e+02  +5.76518566169455e+01  +1.20989426221207e+04

        rx (km)                 ry (km)                 rz (km)               rmag (km)
 +1.50959624245087e+08  +1.45795375300764e+08  +6.27940027604867e+07  +2.19062060599256e+08

        vx (kps)                vy (kps)                vz (kps)              vmag (kps)
 -1.18281591929339e+01  +1.80574267841797e+01  +5.47966628522347e+00  +2.22711192928602e+01

heliocentric coordinates of Mars at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             23:14:53.191
UTC Julian Date      2452997.46867119

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +2.27939358131161e+08  +9.35421253992813e-02  +2.46772248828028e+01  +3.32979306663998e+02

       raan (deg)         true anomaly (deg)        arglat (deg)         period (days)
 +3.37165816846315e+00  +7.03802467689632e+01  +4.33595534329611e+01  +6.86972401908844e+02

        rx (km)                 ry (km)                 rz (km)               rmag (km)
 +1.50959227521675e+08  +1.45798811138344e+08  +6.27945137900982e+07  +2.19064220413627e+08

        vx (kps)                vy (kps)                vz (kps)              vmag (kps)
 -1.66322132870698e+01  +1.68977403329907e+01  +8.19995983019273e+00  +2.50879151864905e+01
```

APPENDIX C

Grazing Flyby Example

This appendix summarizes the trajectory characteristics of a typical grazing flyby simulation. The following is the input data file (`mars03_graze.in`) for this example.

```
*****************************************
** interplanetary trajectory optimization
** script ==> e2m_matlab.m
** Mars grazing flyby mars_graze.in
** March 21, 2020
*****************************************

*******************
* simulation type *
*******************
 1 = minimize departure delta-v
 2 = minimize arrival delta-v
 3 = minimize total delta-v
 4 = no optimization
--------------------
1

departure calendar date initial guess (month, day, year)
6,1,2003

departure date search boundary (days)
30

arrival calendar date initial guess (month, day, year)
12,1,2003

arrival date search boundary (days)
30

***************************
* geocentric phase modeling
***************************

perigee altitude of launch hyperbola (kilometers)
185.32

launch azimuth (degrees)
93.0

launch site latitude (degrees)
28.5

****************************
* encounter planet targeting
****************************

type of targeting
(1 = B-plane, 2 = orbital elements, 3 = EI conditions, 4 = grazing flyby, 5 = node/apse alignment)
4

B dot T
10965.197268

B dot R
-6109.036804

radius of closest approach (kilometers)
5000.0

orbital inclination (degrees)
60.0

EI flight path angle (degrees)
-2.0
```

```
EI altitude (kilometers)
100.0

user-defined b-plane angle (degrees)
-60.0
```

The following is the n-body optimal solution for this example.

```
=======================
optimal n-body solution
=======================

grazing flyby with user-defined b-plane angle

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003
UTC time             14:45:51.038
UTC Julian Date      2452796.11591693

park orbit
----------

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +6.56345780000000e+03  +0.00000000000000e+00  +2.86442848562298e+01  +0.00000000000000e+00

       raan (deg)         true anomaly (deg)        arglat (deg)         period (hrs)
 +2.73567680625535e+00  +1.94687875841345e+02  +1.94687875841345e+02  +1.46996679905864e+00

        rx (km)                ry (km)                rz (km)               rmag (km)
 -6.27202986882620e+03  -1.76187295546076e+03  -7.97761434107232e+02  +6.56345780000000e+03

        vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 +2.28943624327753e+00  -6.51385927168831e+00  -3.61363672257620e+00  +7.79296165049872e+00

departure hyperbola
-------------------

c3                              8.788669  km^2/sec^2

v-infinity                   2964.569009  meters/second

asymptote right ascension     350.006073  degrees

asymptote declination          -6.863099  degrees

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -4.53539009232289e+04  +1.14471649993481e+00  +2.86442848562298e+01  +1.94687875841345e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.73567680625535e+00  +0.00000000000000e+00  +1.94687875841345e+02

        rx (km)                ry (km)                rz (km)               rmag (km)
 -6.27202986882620e+03  -1.76187295546076e+03  -7.97761434107232e+02  +6.56345780000000e+03

        vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 +3.35284518111515e+00  -9.53944960628235e+00  -5.29211700354899e+00  +1.14126759341786e+01

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1063.408938  meters/second
y-component of delta-v      -3025.590335  meters/second
z-component of delta-v      -1678.480281  meters/second

delta-v magnitude            3619.714284  meters/second

transfer time                 201.365951  days

spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------
```

```
UTC calendar date    08-Jun-2003
UTC time             18:20:46.192
UTC Julian Date      2452799.26442352

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -4.56065260606116e+04  +1.14326281915237e+00  +2.85083863319975e+01  +1.94647009750069e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.74800225940308e+00  +1.49479519126339e+02  +3.44126528876407e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +8.99363731026555e+05  -1.79411207101717e+05  -1.20753873976359e+05  +9.25000000000000e+05

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.03057105281273e+00  -5.31560213659139e-01  -3.67298994445695e-01  +3.09868128696913e+00

spacecraft heliocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003
UTC time             18:20:46.192
UTC Julian Date      2452799.26442352

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +1.90700137776633e+08  +2.03948482285259e-01  +2.35007506035249e+01  +2.53585210909749e+02

       raan (deg)         true anomaly (deg)        arglat (deg)          period (days)
 +5.32698298589480e-01  +3.78889347506891e+00  +2.57374104384818e+02  +5.25698598286053e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -3.19300282728087e+07  -1.36202034272512e+08  -5.90928089539813e+07  +1.51863362650590e+08

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.16289249475347e+01  -6.53201165777612e+00  -2.96804426839630e+00  +3.24324737942277e+01

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             23:32:49.201
UTC Julian Date      2452997.48112501

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -5.84868051617740e+03  +1.58068064588750e+00  +7.54855280392424e+00  +3.92440660250177e+01

       raan (deg)          true anomaly (deg)       arglat (deg)
 +1.91320577103909e+02  +4.12165088905013e-06  +3.92440701466686e+01

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -2.16095549688630e+03  -2.60478205801987e+03  +2.82244636178656e+02  +3.39621557972355e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +4.39848761727117e+00  -3.58614490073934e+00  +5.80386417644899e-01  +5.70473285633721e+00

b-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude                 7159.681560  kilometers
b dot r                        0.016827  kilometers
b dot t                     7159.681560  kilometers
b-plane angle                  0.000135  degrees
v-infinity                  2706.056469  meters/second
r-periapsis                 3396.215580  kilometers
decl-asymptote                 7.548553  degrees
rasc-asymptote               281.319552  degrees

flight path angle              0.000003  degrees

spacecraft heliocentric coordinates at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------
```

```
UTC calendar date    23-Dec-2003
UTC time             23:32:49.201
UTC Julian Date      2452997.48112501

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +1.84781911233245e+08  +3.25631781882600e-01  +2.12923804633754e+01  +2.73089275215044e+02

       raan (deg)         true anomaly (deg)        arglat (deg)          period (hrs)
 +3.53851380362963e+02  +1.39047994408028e+02  +5.21372696230724e+01  +1.20340201040916e+04

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +1.50941536653566e+08  +1.45813881287054e+08  +6.28019898359056e+07  +2.19064203809486e+08

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 -1.12994705864271e+01  +1.79866116325784e+01  +6.49793035582642e+00  +2.22130441962354e+01

heliocentric coordinates of Mars at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             23:32:49.201
UTC Julian Date      2452997.48112501

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +2.27939357192010e+08  +9.35421210462794e-02  +2.46772248841462e+01  +3.32979305399218e+02

       raan (deg)         true anomaly (deg)        arglat (deg)          period (days)
 +3.37165817150191e+00  +7.03872827804716e+01  +4.33665881796894e+01  +6.86972397663174e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +1.50941329986410e+08  +1.45816992214685e+08  +6.28033365720434e+07  +2.19066518211216e+08

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 -1.66342637220178e+01  +1.68957596948639e+01  +8.19910677601693e+00  +2.50876618512916e+01
```

APPENDIX D

Node/apse Alignment Example

This appendix summarizes the trajectory characteristics of a typical node/apse alignment scientific simulation. The following is the input data file (`mars03_node-apse.in`) for this example.

```
*********************************
** Earth-to-Mars mission analysis
** script ==> e2m_matlab.m
** Mars '03 mars03_node-apse.in
** March 20, 2020
*********************************

*******************
* simulation type *
*******************
 1 = minimize departure delta-v
 2 = minimize arrival delta-v
 3 = minimize total delta-v
 4 = no optimization
--------------------
1

departure calendar date initial guess (month, day, year)
6,1,2003

departure date search boundary (days)
30

arrival calendar date initial guess (month, day, year)
12,1,2003

arrival date search boundary (days)
30

***************************
* geocentric phase modeling
***************************

perigee altitude of launch hyperbola (kilometers)
185.32

launch azimuth (degrees)
93.0

launch site latitude (degrees)
28.5

****************************
* encounter planet targeting
****************************

type of targeting
(1 = B-plane, 2 = orbital elements, 3 = EI conditions, 4 = grazing flyby, 5 = node/apse alignment)
5

B dot T
10965.197268

B dot R
-6109.036804

radius of closest approach (kilometers)
6000.0

orbital inclination (degrees)
20.0

EI flight path angle (degrees)
-2.0
```

```
EI altitude (kilometers)
100.0

user-defined b-plane angle (degrees)
-60.0
```

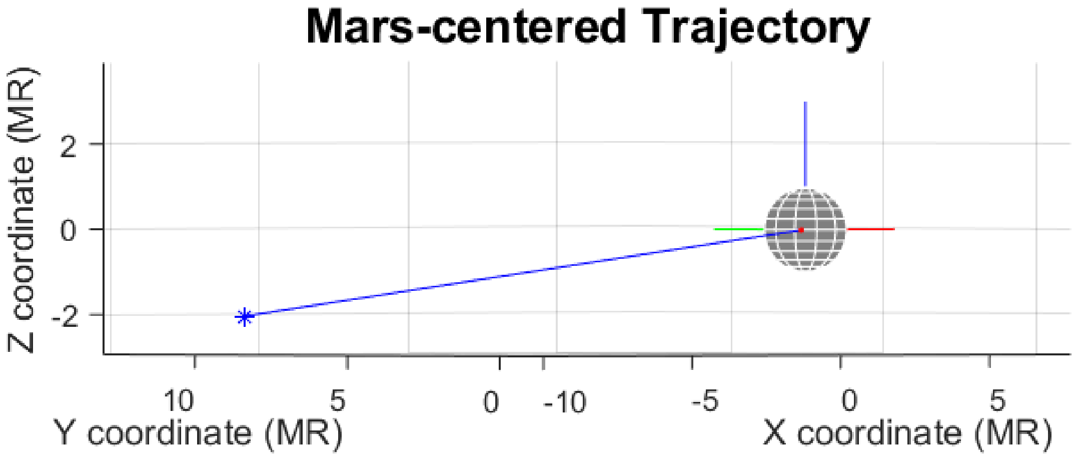
Here is the n-body optimal solution for this example.

```
=======================
optimal n-body solution
=======================

periapsis radius with node/apse alignment

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003
UTC time             14:45:51.038
UTC Julian Date      2452796.11591693

park orbit
----------

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +6.56345780000000e+03  +0.00000000000000e+00  +2.86442848562298e+01  +0.00000000000000e+00

       raan (deg)         true anomaly (deg)        arglat (deg)          period (hrs)
 +2.73502571225612e+00  +1.94682097318372e+02  +1.94682097318372e+02  +1.46996679905864e+00

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27224432182923e+03  -1.76124836785962e+03  -7.97454479185926e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +2.28861115158281e+00  -6.51409622486383e+00  -3.61373223360688e+00  +7.79296165049872e+00

departure hyperbola
-------------------

c3                              8.787703  km^2/sec^2

v-infinity                   2964.406056  meters/second

asymptote right ascension     350.001550  degrees

asymptote declination          -6.865133  degrees

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -4.53588872307019e+04  +1.14470059123402e+00  +2.86442848562298e+01  +1.94682097318372e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.73502571225612e+00  +0.00000000000000e+00  +1.94682097318372e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27224432182923e+03  -1.76124836785962e+03  -7.97454479185925e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.35162441603840e+00  -9.53976123929029e+00  -5.29223725000431e+00  +1.14126336066301e+01

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1063.013264  meters/second
y-component of delta-v      -3025.665014  meters/second
z-component of delta-v      -1678.505016  meters/second

delta-v magnitude            3619.671956  meters/second

transfer time                 201.372786  days
```

```
spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003
UTC time             18:20:58.113
UTC Julian Date      2452799.26456149

        sma (km)              eccentricity        inclination (deg)         argper (deg)
 -4.56115744169914e+04  +1.14324552578020e+00  +2.85084127696153e+01  +1.94641088181904e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.74735996684184e+00  +1.49481016341595e+02  +3.44122104523500e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +8.99345094127006e+05  -1.79482473673027e+05  -1.20786767958654e+05  +9.25000000000000e+05

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.03036351378420e+00  -5.31770640119679e-01  -3.67389880360822e-01  +3.09852519169352e+00

spacecraft heliocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003
UTC time             18:20:58.113
UTC Julian Date      2452799.26456149

        sma (km)              eccentricity        inclination (deg)         argper (deg)
 +1.90697696499531e+08  +2.03938304452635e-01  +2.35007508486982e+01  +2.53582310384728e+02

       raan (deg)         true anomaly (deg)        arglat (deg)           period (days)
 +5.32699039760613e-01  +3.79192460197976e+00  +2.57374234986707e+02  +5.25688503601348e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -3.19297059990725e+07  -1.36202177067911e+08  -5.90928728503408e+07  +1.51863447823801e+08

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.16287318217971e+01  -6.53216063159106e+00  -2.96810847511899e+00  +3.24323213336964e+01

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             23:42:39.791
UTC Julian Date      2452997.48796054

        sma (km)              eccentricity        inclination (deg)         argper (deg)
 -5.84977342875162e+03  +2.02569092778115e+00  +8.68487181617008e+00  +3.59998107189667e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.21181199128523e+02  +5.67882033488072e-06  +3.59998112868487e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -4.51595948946793e+03  -3.95054734403833e+03  -2.98408722449589e-02  +6.00005953544581e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.02477535664439e+00  -3.45769448550182e+00  +7.01740795991967e-01  +4.64729569298020e+00

b-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude                10305.274965  kilometers
b dot r                      774.934833  kilometers
b dot t                    10276.096930  kilometers
b-plane angle                  4.312592  degrees
v-infinity                  2705.803671  meters/second
r-periapsis                 6000.059535  kilometers
decl-asymptote                 7.545610  degrees
rasc-asymptote               281.313442  degrees

flight path angle              0.000004  degrees
```

```
spacecraft heliocentric coordinates at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             23:42:39.791
UTC Julian Date      2452997.48796054

        sma (km)              eccentricity          inclination (deg)         argper (deg)
 +1.82174724055378e+08  +2.93987883664952e-01  +2.22383620120240e+01  +2.64436525957680e+02

       raan (deg)          true anomaly (deg)         arglat (deg)            period (hrs)
 +3.56965529861675e+02  +1.44811967695059e+02  +4.92484936527396e+01  +1.17802289489975e+04

        rx (km)                 ry (km)                 rz (km)                 rmag (km)
 +1.50930794154654e+08  +1.45821510362140e+08  +6.28057948412270e+07  +2.19062971252762e+08

        vx (kps)                vy (kps)                vz (kps)                vmag (kps)
 -1.22471146079017e+01  +1.69895122475576e+01  +6.67174283912272e+00  +2.19806163503295e+01

heliocentric coordinates of Mars at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003
UTC time             23:42:39.791
UTC Julian Date      2452997.48796054

        sma (km)              eccentricity          inclination (deg)         argper (deg)
 +2.27939356676415e+08  +9.35421186568387e-02  +2.46772248848826e+01  +3.32979304704620e+02

       raan (deg)          true anomaly (deg)         arglat (deg)            period (days)
 +3.37165817316802e+00  +7.03911445738126e+01  +4.33704492784325e+01  +6.86972395332290e+02

        rx (km)                 ry (km)                 rz (km)                 rmag (km)
 +1.50931505628363e+08  +1.45826970356238e+08  +6.28081787422398e+07  +2.19067779446090e+08

        vx (kps)                vy (kps)                vz (kps)                vmag (kps)
 -1.66353890134880e+01  +1.68946725035214e+01  +8.19863852063454e+00  +2.50875227996579e+01
```

The following image confirms that the spacecraft is at the ascending node of the areocentric hyperbola at closest approach (red dot).



At this point the spacecraft's argument of latitude is approximately zero degrees and the right ascension of the ascending node (RAAN) is `221.18` degrees.

Note also the geocentric speed of the spacecraft at the Earth SOI is 3098 meters/second compared to the n-body v-infinity value of 2964 meters/second. A difference of 134 meters/second. Of course, 925,000 kilometers is not quite infinity.

Typically, a retrograde propulsive maneuver is performed at periapsis to create an elliptical orbit about Mars. At the next or any subsequent apoapsis, an efficient plane change maneuver can be performed to establish the orbital inclination of the mission orbit. Finally, one or more propulsive maneuvers at periapsis will establish a circular or elliptical mission orbit.

The velocity decrement required to establish an elliptical *capture* orbit of eccentricity *e* is given by

$$\Delta v = \sqrt{v_\infty^2 + \frac{2\mu}{r_p}} - \sqrt{\frac{\mu(1+e)}{r_p}} = \sqrt{v_\infty^2 + \frac{2\mu}{r_p}} - \sqrt{\frac{2\mu r_a}{r_p\left(r_a + r_p\right)}}$$

where r_p and r_a are the periapsis and apoapsis radii of the capture orbit.

For a circular capture orbit of radius *r*, the impulsive delta-v equation simplifies to

$$\Delta v = \sqrt{v_\infty^2 + \frac{2\mu}{r}} - \sqrt{\frac{\mu}{r}} = \sqrt{v_\infty^2 + \frac{2\mu}{r}} - v_{lc}$$

where v_{lc} is the "local circular" speed of the capture orbit.

If we set the partial derivative $\partial \Delta v / \partial r_a = 0$ from the elliptical orbit capture expression, we find a "best" capture apoapsis radius and corresponding minimum delta-v according to

$$r_a = \frac{2\mu}{v_\infty^2} \qquad \Delta v = v_\infty \sqrt{\frac{1-e}{2}}$$

where $e = r_a - r_p / r_a + r_p$ is the orbital eccentricity of the elliptical capture orbit. Notice the optimum apoapsis radius is dependent only on the incoming v-infinity and the planet's gravity.

Likewise, the optimum capture periapsis radius and minimum capture delta-v is

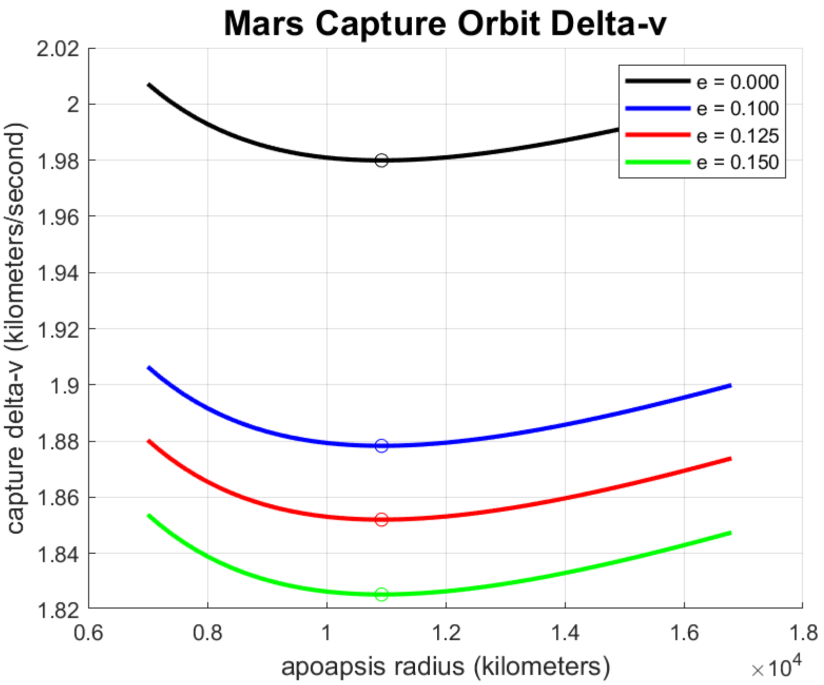
$$r_p = \frac{2\mu}{v_\infty^2}\left[\frac{1-e}{1+e}\right] \qquad \Delta v = \sqrt{v_\infty^2 + \frac{2\mu}{r_p}} - \sqrt{\frac{\mu(1+e)}{r_p}}$$

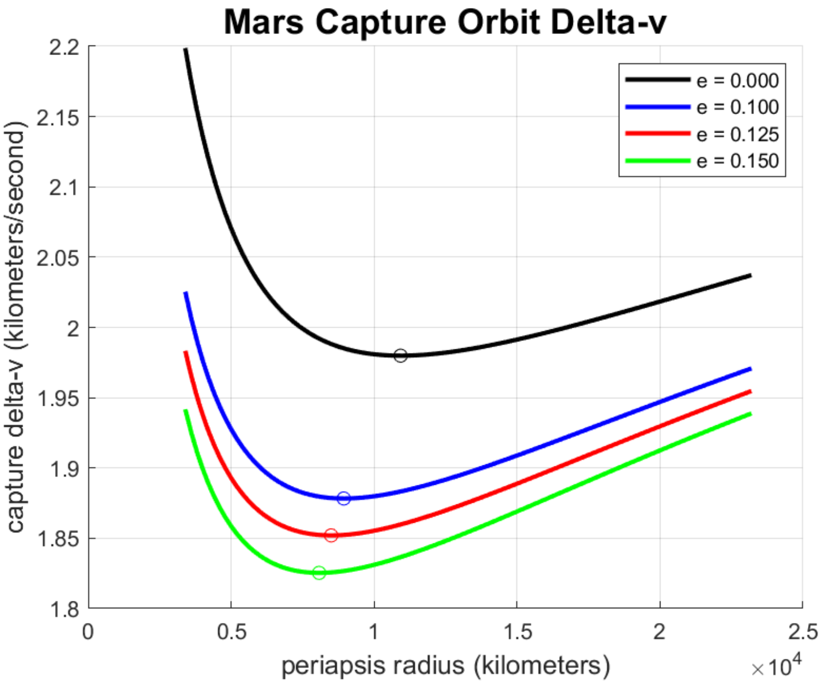
For a circular capture orbit, the optimum periapsis and apoapsis radii are identical and the minimum delta-v is as follows

$$r_a = r_p = \frac{2\mu}{v_\infty^2} \qquad \Delta v = \sqrt{v_\infty^2 + \frac{2\mu}{r}} - \sqrt{\frac{\mu}{r}}$$

The following two plots illustrate the behavior of capture delta-v as a function of the eccentricity of the capture orbit. Each line of the graphs is labeled with a small circle to indicate the optimum radius and impulsive capture delta-v. The v-infinity for this example is 2.8 kilometers meters/second.

In the first graph we can see that the optimum apoapsis radius is independent of orbital eccentricity.

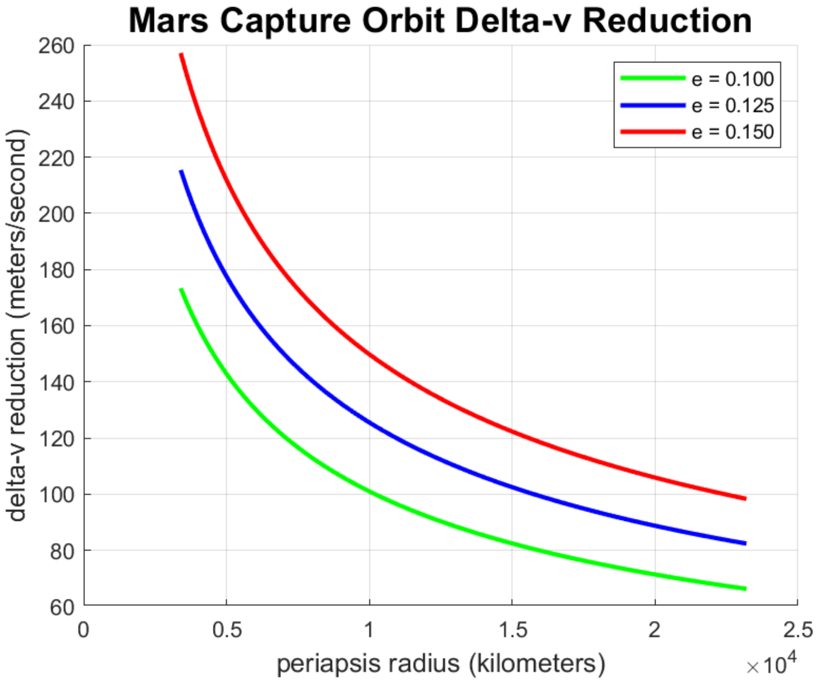




The reduction in delta-v required for capture between a circular and elliptical orbit is given by

$$\Delta\left(\Delta v\right) = v_{lc}\left(\sqrt{1+e} - 1\right)$$

where v_{lc} is the local circular velocity evaluated at the periapsis of the capture orbit and e is the orbital eccentricity of the capture orbit. The following is a graph of the impulsive delta-v reduction in meters/second as a function of the capture orbit periapsis radius and orbital eccentricity.



As an exercise, see if you can derive the expression for the optimum circular orbit radius r_c from

$$\frac{\partial}{\partial r_c}\Delta v = -\left(v_\infty^2 + \frac{2\mu}{r_c}\right)^{-1/2} \mu r_c^{-2} + \frac{1}{2}\sqrt{\mu} r_c^{-3/2} = 0$$

Hint: multiply the expression by $r_c^{3/2}$, rearrange and square the result. Solve for r_c.

Questions: are these equations and results valid as well for the Earth departure hyperbolic trajectory? Is there a reasonable optimal circular orbit altitude that minimizes the interplanetary injection delta-v?

APPENDIX E

Verification of the Optimal Solution

After the n-body optimization has finished, the `e2m_matlab` script will verify the solution by integrating the trajectory from interplanetary injection to closest approach at Mars. For these calculations the script uses an RKF78 algorithm and the same geocentric and heliocentric equations of motion as the optimal n-body calculations.

The integration begins with the n-body solution for the state vector at injection. The software then integrates the spacecraft motion to the n-body predicted time of exit from the Earth's sphere-of-influence (SOI). The final integration is from the SOI to closest approach at Mars.

At each event the MATLAB script will display orbital elements and state vector information. The following is the verification output for a typical trajectory.

```
====================================
verification of the optimal solution
====================================

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date         05-Jun-2003
UTC time                  14:45:51.038
UTC Julian Date           2452796.11591693

park orbit
----------

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 +6.56345780000000e+03  +0.00000000000000e+00  +2.86442848562298e+01  +0.00000000000000e+00

       raan (deg)         true anomaly (deg)        arglat (deg)          period (hrs)
 +2.67612391703511e+00  +1.94740980977338e+02  +1.94740980977338e+02  +1.46996679905864e+00

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27207315457118e+03  -1.76043889495244e+03  -8.00581996566851e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +2.28956784023919e+00  -6.51430084250126e+00  -3.61275724682320e+00  +7.79296165049872e+00

departure hyperbola
-------------------

c3                              8.789418  kilometers^2/second^2

v-infinity                   2964.695346  meters/second

asymptote right ascension     349.992812  degrees

asymptote declination          -6.838784  degrees

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -4.53500355907832e+04  +1.14472883459730e+00  +2.86442848562298e+01  +1.94740980977338e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.67612391703511e+00  +0.00000000000000e+00  +1.94740980977338e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27207315457118e+03  -1.76043889495244e+03  -8.00581996566853e+02  +6.56345780000000e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.35304754485751e+00  -9.54012371353455e+00  -5.29084423869317e+00  +1.14127087523397e+01
```

```
spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date         08-Jun-2003
UTC time                  18:20:36.728
UTC Julian Date           2452799.26431399

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -4.56025217169717e+04  +1.14327373026335e+00  +2.85069555886794e+01  +1.94699397013259e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +2.68905716456125e+00  +1.49478601382173e+02  +3.44177998395432e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +8.99372529799998e+05  -1.79627063185499e+05  -1.20366817366405e+05  +9.24999999951575e+05

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.03072155847099e+00  -5.32311681809966e-01  -3.66011314434666e-01  +3.09880512034339e+00

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

UTC calendar date         23-Dec-2003
UTC time                  22:42:46.653
UTC Julian Date           2452997.44637330

        sma (km)             eccentricity        inclination (deg)         argper (deg)
 -5.84498024594246e+03  +1.85543474434185e+00  +6.00000257076313e+01  +1.13895639725033e+02

       raan (deg)          true anomaly (deg)       arglat (deg)
 +1.05732418363297e+02  +7.42999176171718e-05  +1.13895714024950e+02

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -1.65091442186080e+03  -2.56925104018060e+03  +3.95896735121172e+03  +4.99999918237368e+03

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +2.19016127142701e+00  -4.08068180918405e+00  -1.73492370807993e+00  +4.94557688197466e+00

b-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude                 9135.085370  kilometers
b dot r                    -7888.072112  kilometers
b dot t                     4607.396563  kilometers
b-plane angle                300.289058  degrees
v-infinity                  2706.912893  meters/second
r-periapsis                 4999.999182  kilometers
decl-asymptote                 7.541853  degrees
rasc-asymptote               281.348532  degrees

flight path angle              0.000048  degrees
```

For this used-defined periapsis radius and orbital inclination example, we see that the geocentric distance at exit from the Earth's SOI is `924,999.999951575` kilometers. The areocentric flight path angle at closest approach is nearly zero degrees, the periapsis radius is `4999.99918237368` kilometers and the orbital inclination is `60.0000257076313` degrees. The radius and inclination "targets" are 5000 kilometers and 60 degrees, respectively.

APPENDIX F

Optimization Toolbox Implementation

There is a version of this MATLAB script named `e2m_matlab_otb` that uses the Mathworks Optimization Toolbox to solve this orbital mechanics problem. This appendix describes the source code implementation using the `fmincon/interior-point` algorithm. Unlike SNOPT, this version requires the mission constraints and objective algorithms two different MATLAB functions.

The following source code solves the optimal two-body Lambert problem. The MATLAB function `twobody_objective` evaluates the Lambert solution and return the proper impulsive delta-v objective function. No mission constraints function is required for this part of the simulation.

```
%---------------------------------------
% find optimal two-body lambert solution
%---------------------------------------

xg(1) = jdtdb_depart - jdtdb0;

xg(2) = jdtdb_arrive - jdtdb0;

xg = xg';

% bounds on control variables

xlwr(1) = xg(1) - ddays1;

xupr(1) = xg(1) + ddays1;

xlwr(2) = xg(2) - ddays2;

xupr(2) = xg(2) + ddays2;

% find optimum

options = optimoptions('fmincon', 'Algorithm', 'interior-point','MaxFunctionEvaluations', 500);

[x, ~] = fmincon('twobody_objective', xg, [], [], [], [], xlwr, xupr, [], options);

jdtdb_depart = x(1) + jdtdb0;

jdtdb_arrive = x(2) + jdtdb0;

% transfer time (days)

taud = jdtdb_arrive - jdtdb_depart;
```

This part of the source code solves the n-body trajectory optimization. For this part of the simulation, the MATLAB objective function is `nbody_objective.m` and the mission constraints function is called `nbody_contraints.m`.

```
% initial guess for launch vinf, rla and dla

xg(1) = norm(twobody_dv1);

xg(2) = twobody_rasc1;

xg(3) = twobody_decl1;

% define lower and upper bounds for vinf, rla and dla

xlwr(1) = xg(1) - 0.05;
xupr(1) = xg(1) + 0.05;
```

```
xlwr(2) = xg(2) - 10.0 * dtr;
xupr(2) = xg(2) + 10.0 * dtr;

xlwr(3) = xg(3) - 1.0 * dtr;
xupr(3) = xg(3) + 1.0 * dtr;

% define optimization options

options = optimoptions('fmincon', 'Display', 'iter', 'Algorithm', 'interior-point', ...
    'MaxFunctionEvaluations', 500, 'FiniteDifferenceType', 'forward');

% optimize with user-defined mission constraints

[x, fval] = fmincon('nbody_objective', xg, [], [], [], [], xlwr, xupr, 'nbody_constraints',
options);
```

Feel free to experiment with other `fmincon` non-linear programming algorithms such as *sqp*, etc.