Osculating Orbital Elements of the Moon

This document describes a MATLAB function that calculates the osculating classical orbital elements of the Moon in the mean ecliptic and mean equinox of date coordinate system. It is based on the book *Lunar Tables and Programs From 4000 B.C. TO A.D. 8000* by Michelle Chapront-Touze and Jean Chapront. This book and its optional companion software are available from Willmann-Bell at www.willbell.com.

The fundamental time argument of this algorithm is

$$t = \frac{JED - 2451545}{36525}$$

where *JED* is the Julian Date on the terrestrial time (TDT or TT) scale.

The osculating orbital elements are calculated from series of the form

$$a = 383397.6 + S_a + tS_a''$$

where

$$S_a = \sum_{n=1}^{30} a_n \cos\left(\xi_n^{(0)} + \xi_n^{(1)} t + \xi_n^{(2)} 10^{-4} t^2 + \xi_n^{(3)} 10^{-6} t^3 + \xi_n^{(4)} 10^{-8} t^4\right)$$

$$S_a'' = \sum_{n=1}^{3} a_n'' \cos\left(\xi_n''^{(0)} + \xi_n''^{(1)} t\right)$$

The first few multipliers and trigonometric arguments for this orbital element are

n	a_n	$\xi_n^{(0)}$	$\xi_n^{(1)}$	$\xi_n^{(2)}$	$\xi_n^{(3)}$	$\xi_n^{(4)}$
1	3400.4	235.7004	890534.2230	-32.601	3.664	-1.769
2	-635.6	100.7370	413335.3554	-122.571	-10.684	5.028

n	a_n''	$\xi_n''^{(0)}$	$\xi_n''^{(1)}$
1	-0.55	238.2	854535.2
2	0.10	103.2	377336.3

The syntax for this MATLAB function is

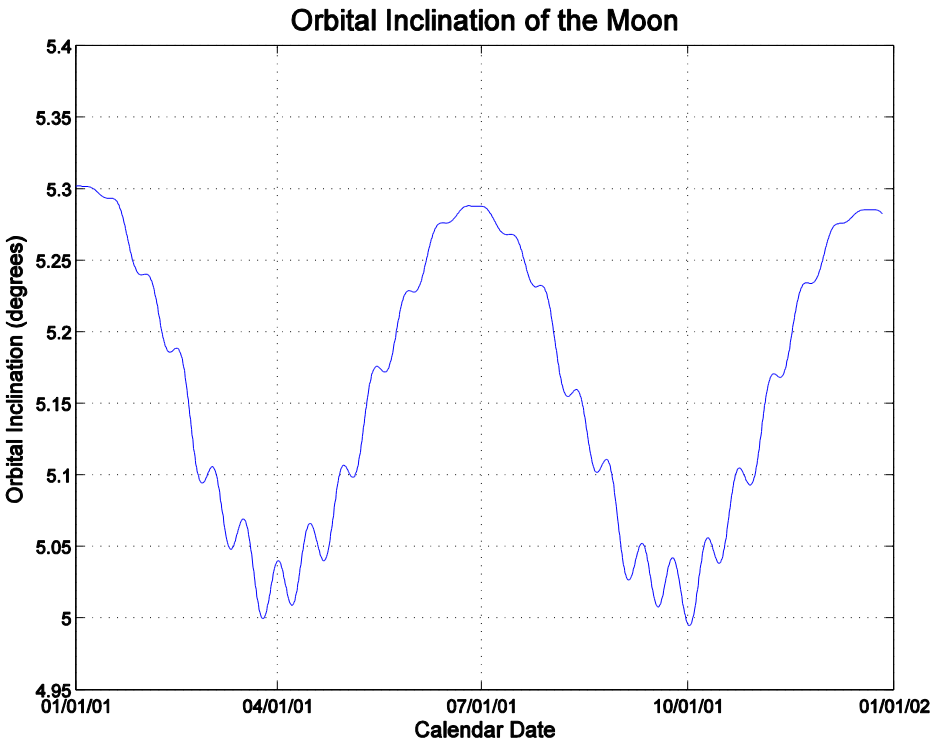
```
function oev = elp2000(tjd)

% osculating orbital elements of the moon

% mean ecliptic and mean equinox of date

% input

%  tjd = tdt Julian date
```

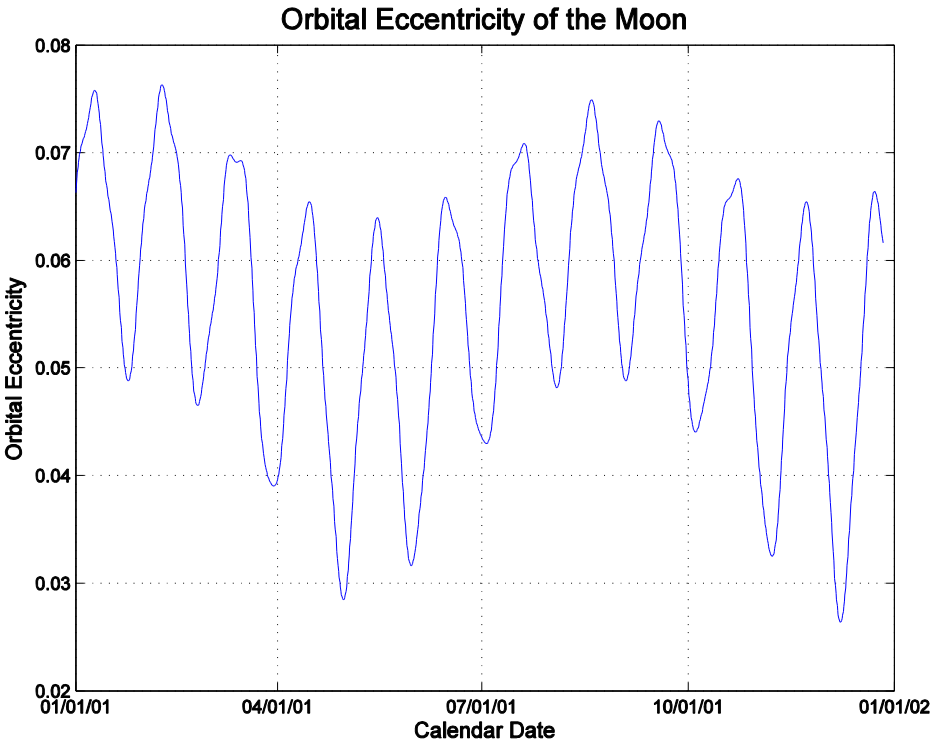
```
% output

%  oev(1)  = semimajor axis (kilometers)
%  oev(2)  = orbital eccentricity (non-dimensional)
%            (0 <= eccentricity < 1)
%  oev(3)  = orbital inclination (radians)
%            (0 <= inclination <= pi)
%  oev(4)  = argument of perigee (radians)
%            (0 <= argument of perigee <= 2 pi)
%  oev(5)  = right ascension of ascending node (radians)
%            (0 <= raan <= 2 pi)
%  oev(6)  = true anomaly (radians)
%            (0 <= true anomaly <= 2 pi)
```

This software suite includes a script called `demo_elp` that demonstrates how to interact with this MATLAB function. The following is a typical user interaction with this script.

```
program demo_elp

< orbital elements of the moon >

please input a UTC calendar date

please input the calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
? 4,1,2013

please input the simulation period (days)
? 180

please input the graphics step size (days)
? 0.1

please select the orbital element to plot

  <1> semimajor axis
  <2> eccentricity
  <3> orbital inclination
  <4> argument of perigee
  <5> right ascension of the ascending node
  <6> true anomaly
  <7> apogee radius
  <8> perigee radius
  <9> geocentric distance

? 9
```

The following are two graphics displays created for this example. The first plot is the geocentric orbital inclination of the moon and the second display is the orbital eccentricity of the moon's orbit.


Orbital Inclination of the Moon
Orbital Inclination (degrees)
5.4
5.35
5.3
5.25
5.2
5.15
5.1
5.05
5
4.95
01/01/01
04/01/01
07/01/01
10/01/01
01/01/02
Calendar Date


Orbital Eccentricity of the Moon
Orbital Eccentricity
0.08
0.07
0.06
0.05
0.04
0.03
0.02
01/01/01
04/01/01
07/01/01
10/01/01
01/01/02
Calendar Date