A MATLAB Implementation of the IAU 1980 Nutation Theory

This document describes a MATLAB function and companion demonstration script that calculates the nutation in longitude and obliquity based on the IAU 1980 theory. This nutation theory was developed by Wahr and is based on work by Kinoshita and Gilbert and Dziewonski.

This function requires initialization the first time it is called. This can be accomplished by placing the following statement in the main script along with a `global inutate` statement.

```
inutate = 1;
```

This MATLAB function also requires the comma-separated data file named `nut80.csv`.

The nutation in longitude is determined from a series of the form

$$\Delta\psi = \sum_{i=1}^{n} S_i \sin A_i$$

Likewise, the nutation in obliquity is determined from

$$\Delta\varepsilon = \sum_{i=1}^{n} C_i \cos A_i$$

where

$$A_i = a_i l + b_i l' + c_i F + d_i D + e_i \Omega$$

and l, l', F, D and Ω are fundamental arguments.

The nutation matrix defined by

$$\mathbf{N} = \begin{bmatrix} \cos\Delta\psi & -\sin\Delta\psi\cos\varepsilon_0 & -\sin\Delta\psi\sin\varepsilon_0 \\ \sin\Delta\psi\cos\varepsilon & \cos\Delta\psi\cos\varepsilon\cos\varepsilon_0 + \sin\varepsilon\sin\varepsilon_0 & \cos\Delta\psi\cos\varepsilon\cos\varepsilon_0 - \sin\varepsilon\cos\varepsilon_0 \\ \sin\Delta\psi\sin\varepsilon & \cos\Delta\psi\sin\varepsilon\cos\varepsilon_0 - \cos\varepsilon\sin\varepsilon_0 & \cos\Delta\psi\sin\varepsilon\sin\varepsilon_0 + \cos\varepsilon\cos\varepsilon_0 \end{bmatrix}$$

can be used to transform a mean equinox of date position vector $\mathbf{r}_0$ to a true equinox of date position vector $\mathbf{r}$ as follows:

$$\mathbf{r} = [\mathbf{N}]\mathbf{r}_0$$

In this matrix ε_0 is the mean obliquity of the ecliptic and $\varepsilon = \varepsilon_0 + \Delta\varepsilon$ is the true obliquity.

The nutation matrix can also be expressed as a combination of individual rotations according to

$$\mathbf{N} = \mathbf{R}_1(-\varepsilon)\mathbf{R}_3(-\Delta\psi)\mathbf{R}_1(+\varepsilon_0)$$

where

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \qquad R_3(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The mean obliquity of the ecliptic is calculated from

$$\varepsilon_0 = 23^0 26' 21.''448 - 46.''8150T - 0.''00059T^2 + 0.''001813T^3$$

where $T = (JD - 2451545.0)/36525$ and JD is the Julian Date.

If second-order terms are neglected, a linearized nutation matrix can be calculated from

$$\mathbf{N} = \begin{bmatrix} 1 & -\Delta\psi\cos\varepsilon & -\Delta\psi\sin\varepsilon \\ \Delta\psi\cos\varepsilon & 1 & -\Delta\varepsilon \\ \Delta\psi\sin\varepsilon & \Delta\varepsilon & 1 \end{bmatrix}$$

Mean equinox equatorial rectangular position coordinates can be converted to true equinox coordinates by adding the following corrections to the respective components:

$$\Delta r_x = -\left(r_y \cos\varepsilon + r_z \sin\varepsilon\right)\Delta\psi$$

$$\Delta r_y = r_x \cos\varepsilon\,\Delta\psi - r_z\Delta\varepsilon$$

$$\Delta r_z = r_x \sin\varepsilon\,\Delta\psi + r_y\Delta\varepsilon$$

Finally, the nutation corrections to right ascension and declination are given by

$$\Delta\alpha = \left(\cos\varepsilon + \sin\varepsilon\sin\alpha\tan\delta\right)\Delta\psi - \cos\alpha\tan\delta\,\Delta\varepsilon$$

$$\Delta\delta = \sin\varepsilon\cos\alpha\,\Delta\psi + \sin\alpha\,\Delta\varepsilon$$

The syntax of this MATLAB function is

```
function [dpsi, deps] = nut80(jdate)

% this function evaluates the nutation series and returns the
% values for nutation in longitude and nutation in obliquity.
% wahr nutation series for axis b for gilbert & dziewonski earth
% model 1066a. see seidelmann (1982) celestial mechanics 27,
% 79-106. 1980 iau theory of nutation.

% jdate = tdb julian date (in)
% dpsi  = nutation in longitude in arcseconds (out)
% deps  = nutation in obliquity in arcseconds (out)
```

The first few terms of the `nut80.csv` data file are as follows

```
  0.,  0.,  0.,  0.,  1., -171996., -174.2,  92025.,  8.9
  0.,  0.,  2., -2.,  2.,  -13187.,   -1.6,   5736., -3.1
  0.,  0.,  2.,  0.,  2.,   -2274.,   -0.2,    977., -0.5
  0.,  0.,  0.,  0.,  2.,    2062.,    0.2,   -895.,  0.5
  0.,  1.,  0.,  0.,  0.,    1426.,   -3.4,     54., -0.1
  1.,  0.,  0.,  0.,  0.,     712.,    0.1,     -7.,  0.0
  0.,  1.,  2., -2.,  2.,    -517.,    1.2,    224., -0.6
  0.,  0.,  2.,  0.,  1.,    -386.,   -0.4,    200.,  0.0
  1.,  0.,  2.,  0.,  2.,    -301.,    0.0,    129., -0.1
  0., -1.,  2., -2.,  2.,     217.,   -0.5,    -95.,  0.3
  1.,  0.,  0., -2.,  0.,    -158.,    0.0,     -1.,  0.0
  0.,  0.,  2., -2.,  1.,     129.,    0.1,    -70.,  0.0
 -1.,  0.,  2.,  0.,  2.,     123.,    0.0,    -53.,  0.0
  1.,  0.,  0.,  0.,  1.,      63.,    0.1,    -33.,  0.0
```

The `demo_nut80.zip` archive includes a script called `demo_nut80` that demonstrates how to interact with this MATLAB function. The following is a typical user interaction with this script.

```
demo_nut80 - demonstrates how to use the nut80.m function

please input a UTC calendar date and time

please input the calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
? 12,28,2012

please input the universal time
(0 <= hours <= 24, 0 <= minutes <= 60, 0 <= seconds <= 60)
? 12,14,51
```

The following is the script output for this example.

```
program demo_nut80

UTC calendar date       28-Dec-2012

UTC Julian date          2456290.01031250

TDB Julian date          2456290.01107852


nutation in obliquity       -6.05166231 arc seconds

nutation in longitude       14.45064304 arc seconds
```

The following are the results for this same calendar year and time using the Multiyear Interactive Computer Almanac (MICA) published by the United States Naval Observatory.

```
                          NUTATION AND OBLIQUITY

   Date         Time         Obliq. of Ecliptic          Nutation in
        (UT1)                      Mean     True         Long.      Obliq.
              h  m   s       °  '    "        "            "          "
2012 Dec 28 12:14:51.0     23 26  15.3214  9.2713    +14.4557   - 6.0501
```