triaxial.m – geodetic altitude to a triaxial ellipsoid

This MATLAB function calculates the geodetic altitude relative to a triaxial ellipsoidal planet. The algorithm is based on the numerical method described in "Geodetic Altitude to a Triaxial Ellipsoidal Planet", by Charles C. H. Tang, *The Journal of the Astronautical Sciences*, Vol. 36, No. 3, July-September 1988, pp. 279-283.

This function solves for the real root of the following nonlinear equation:

$$f\left(z_p\right)=\left\{1+\frac{c_y}{\left[c_z+\left(b^2-c^2\right)z_p\right]^2}+\frac{c_x}{\left[c_z+\left(a^2-c^2\right)z_p\right]^2}\right\}z_p^2-c^2=0$$

where

$$c_x=\left(acx_s\right)^2 \qquad c_y=\left(bcy_s\right)^2 \qquad c_z=c^2z_s$$

$a,b,c=$ semi-axes $\left(a \ge b > c\right)$

$x_s, y_s, z_s =$ geocentric coordinates of satellite

$z_p =$ z coordinate of subpoint

The bracketing interval used during the root-finding is $z_{p0}-10 \le z_p \le z_{p0}+10$ (kilometers) where

$$z_{p0}=\frac{z_s}{\sqrt{\left(x_s/a\right)^2+\left(y_s/b\right)^2+\left(z_s/c\right)^2}}$$

The syntax of this MATLAB function is

```
function alt = triaxial(rsc)

% geodetic altitude relative
% to a triaxial ellipsoid

% input

%  rsc = geocentric position vector (km)

% output

%  alt = geodetic altitude (km)
```

The semi-axes in this function are "hardwired" to the values $a=6378.138$ and $b=6367$ (kilometers), and the flattening factor is $f=1/298.257$. These are representative values for the Earth and can be easily changed for other planets or the Moon.

This software suite contains a MATLAB script named `demo_triaxial` that demonstrates how to interact with this function. The following is the output created with this script.

```
program demo_triaxial

altitude relative to a triaxial ellipsoid
```

```
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 altitude =   1519.73117837 kilometers

          sma (km)                eccentricity          inclination (deg)         argper (deg)
  +8.00000000000000e+003  +1.50000000000000e-002  +2.85000000000000e+001  +1.20000000000000e+002

         raan (deg)          true anomaly (deg)         arglat (deg)              period (min)
  +4.50000000000000e+001  +3.00000000000000e+001  +1.50000000000000e+002  +1.18684693004297e+002
```

For this example, the classical orbital elements of the spacecraft are as follows,

semimajor axis = 8000 kilometers

orbital eccentricity = 0.015

orbital inclination = 28.5 degrees

argument of perigee = 120 degrees

RAAN = 45 degrees

true anomaly = 30 degrees