Single Impulse De-orbit

This document describes two MATLAB scripts that be used to compute the characteristics of single impulse de-orbit from Earth orbits. Scripts are provided for calculating the impulsive maneuver required to de-orbit from both circular and elliptical orbits.

cdeorbit.m – single impulse de-orbit from a circular orbit

This MATLAB script calculates the single impulsive maneuver required to establish a reentry altitude and flight path angle relative to a non-rotating, spherical Earth. The algorithm uses a tangential delta-v applied opposite to the velocity vector of an initial circular orbit to establish the de-orbit trajectory.

The scalar magnitude of the single impulsive maneuver required to de-orbit a spacecraft from an initial circular orbit can be determined from the following expression

$$\Delta V = V_{c_e}\sqrt{\frac{1}{\tilde{r}}}\left\{1-\sqrt{\frac{2\left(\tilde{r}-1\right)}{\left(\dfrac{\tilde{r}}{\cos\gamma_e}\right)^2-1}}\right\} = V_{c_i}\left\{1-\sqrt{\frac{2\left(\tilde{r}-1\right)}{\left(\dfrac{\tilde{r}}{\cos\gamma_e}\right)^2-1}}\right\}$$

where

$$\tilde{r} = \frac{h_i + r_{eq}}{h_e + r_{eq}} = \frac{r_i}{r_e} = \text{radius ratio}$$

$$V_{c_e} = \sqrt{\frac{\mu}{\left(h_e + r_{eq}\right)}} = \sqrt{\frac{\mu}{r_e}} = \text{ local circular velocity at entry interface}$$

$$V_{c_i} = \sqrt{\frac{\mu}{\left(h_i + r_{eq}\right)}} = \sqrt{\frac{\mu}{r_i}} = \text{ local circular velocity of initial circular orbit}$$

and

$\gamma_e =$ flight path angle at entry interface

$h_i =$ altitude of initial circular orbit

$h_e =$ altitude at entry interface

$r_i =$ radius of initial circular orbit

$r_e =$ radius at entry interface

$r_{eq} =$ Earth equatorial radius

$\mu =$ Earth gravitational constant

This algorithm is described in the technical article, "Deboost from Circular Orbits", A. H. Milstead, *The Journal of the Astronautical Sciences*, Vol. XIII, No. 4, pp. 170-171, Jul-Aug., 1966. Additional information can be found in Chapter 5 of *Hypersonic and Planetary Entry Flight Mechanics* by Vinh, Busemann and Culp, The University of Michigan Press.

The true anomaly on the de-orbit trajectory at the entry interface θ_e can be determined from the following two equations

$$\sin\theta_e = \frac{\dot{r}}{e_d}\sqrt{\frac{a_d\left(1-e_d^2\right)}{\mu}}$$

$$\cos\theta_e = \frac{a_d\left(1-e_d^2\right)}{e_d r_e} - \frac{1}{e_d}$$

and the following four quadrant inverse tangent operation

$$\theta_e = \tan^{-1}\left(\sin\theta_e, \cos\theta_e\right)$$

where

$e_d =$ eccentricity of the de-orbit trajectory

$a_d =$ semimajor axis of the de-orbit trajectory

$$\dot{r} = -\sqrt{\frac{\mu\left[2a_d r_e - r_e^2 - a_d^2\left(1-e_d^2\right)\right]}{a_d r_e^2}}$$

The elapsed time-of-flight between perigee of the de-orbit trajectory and the entry true anomaly θ_e is given by

$$t\left(\theta_e\right) = \frac{\tau}{2\pi}\left[2\tan^{-1}\left\{\sqrt{\frac{1-e_d}{1+e_d}}\tan\frac{\theta_e}{2}\right\} - \frac{e_d\sqrt{1-e_d^2}\sin\theta_e}{1+e_d\cos\theta_e}\right]$$

In this equation τ is the Keplerian orbital period of the de-orbit trajectory and is equal to $2\pi\sqrt{a_d^3/\mu}$.

Therefore, the flight time between the de-orbit impulse and entry interface is given by

$$\Delta t = t\left(\theta_e\right) - t\left(180^\circ\right) = t\left(\theta_e\right) - \frac{\tau}{2}$$

Finally, the orbital speed at the entry interface V_e can be determined from

$$V_e = \sqrt{\frac{2\mu}{r_e} - \frac{\mu}{a_d}}$$

This MATLAB script will prompt you for the altitude of the initial circular orbit, and the entry altitude and flight path angle. The following is a typical user interaction with this script.

```
             program cdeorbit

< single impulse deorbit from circular orbits >


please input the initial altitude (kilometers)
? 1000

please input the entry altitude (kilometers)
? 100

please input the entry flight path angle (degrees)
? -2
```

The following is the script output created for this example.

```
             program cdeorbit

< single impulse deorbit from circular orbits >


initial altitude      1000.000000  kilometers

entry altitude         100.000000  kilometers

entry fpa               -2.000000  degrees


entry trajectory

semimajor axis              6896.07935765  kilometers

eccentricity                   0.06990358

argument of perigee          180.00000000  degrees

perigee altitude              35.87871531  kilometers

apogee altitude             1000.00000000  kilometers

entry true anomaly           328.04948058  degrees

entry velocity              8078.31275892  meters/second

impulse-to-entry time         40.13350666  minutes

deorbit delta-v              261.55416617  meters/second
```

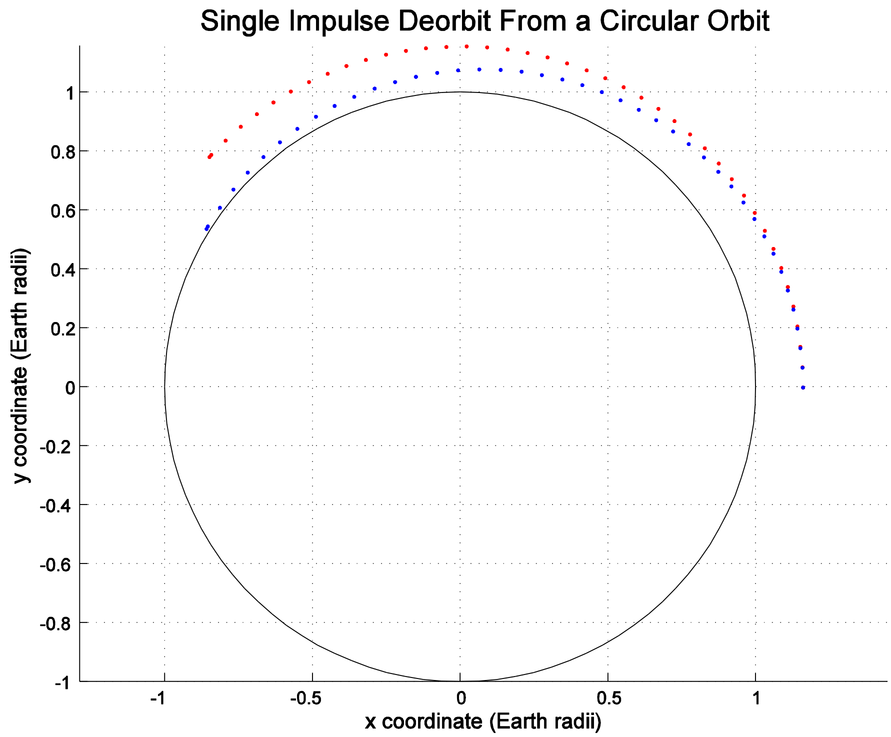
The software will also calculate and display the entry velocity and flight path angle relative to a rotating spherical Earth. The following is the relative flight information for this example.

```
relative flight path coordinates

flight path angle            -2.12418719  degrees

velocity magnitude            7.60622497  kilometers/second
```

Finally, the software will graphically display the initial circular orbit and the de-orbit trajectory. The graphic display for this example is as follows where the red dots represent the original circular orbit and the blue dots represent the de-orbit trajectory, both at one minute intervals. The black circle is the surface of a spherical Earth and the distances are in Earth radii.



The maneuver creates an elliptical de-orbit trajectory with an apogee located at the maneuver point. The apogee altitude of this trajectory is equal to the altitude of the initial circular orbit.

edeorbit.m – single impulse de-orbit from an elliptical orbit

This MATLAB script calculates the single impulsive maneuver required to establish a reentry altitude and flight path angle relative to a non-rotating spherical Earth. The algorithm uses a tangential ΔV applied opposite to the velocity vector at apogee of the initial elliptical orbit to establish the de-orbit trajectory that enters the Earth's atmosphere.

The scalar magnitude of this de-orbit delta-v is given by

$$\Delta V = \sqrt{\frac{\mu}{r_e}}\left(\sqrt{\frac{2\tilde{r}_p}{\tilde{r}_a\left(\tilde{r}_a+\tilde{r}_p\right)}} - \sqrt{\frac{2\left(\tilde{r}_a-1\right)}{\tilde{r}_a\left(\tilde{r}_a^2-\cos^2\gamma_e\right)}}\cos\gamma_e\right)$$

where

$r_e =$ geocentric radius at the entry altitude

$\tilde{r}_a = r_a / r_e$

$\tilde{r}_p = r_p / r_e$

$\gamma_e =$ flight path angle at entry

$r_a =$ apogee radius of the initial elliptical orbit

$r_p =$ perigee radius of the initial elliptical orbit

$\mu =$ gravitational constant of the Earth

The true anomaly at entry can be determined from the following series of equations:

$$\sin\theta_e = \frac{\dot{r}}{e_d}\sqrt{\frac{a_d\left(1-e_d^2\right)}{\mu}}$$

$$\cos\theta_e = \frac{a_d\left(1-e_d^2\right)}{e_d r_e} - \frac{1}{e_d}$$

$$\theta_e = \tan^{-1}\left(\sin\theta_e, \cos\theta_e\right)$$

where

$e_d =$ eccentricity of deorbit trajectory

$a_d =$ semimajor axis of deorbit trajectory

$$\dot{r} = -\sqrt{\frac{\mu\left[2a_d r_e - r_e^2 - a_d^2\left(1-e_d^2\right)\right]}{a_d r_e^2}}$$

and the inverse tangent is a four quadrant operation.

The time of flight between perigee and the entry true anomaly θ_e is given by:

$$t\left(\theta_e\right) = \frac{\tau}{2\pi}\left[2\tan^{-1}\left\{\sqrt{\frac{1-e_d}{1+e_d}}\tan\frac{\theta_e}{2}\right\} - \frac{e_d\sqrt{1-e_d^2}\sin\theta_e}{1+e_d\cos\theta_e}\right]$$

In this equation τ is the orbital period of the de-orbit trajectory.

Therefore, the flight time between the de-orbit impulse time and entry is given by

$$\Delta t = t\left(\theta_e\right) - t\left(180^\circ\right) = t\left(\theta_e\right) - \frac{\tau}{2}$$

Finally, the speed at reentry V_e can be determined from

$$V_e = \sqrt{\frac{2\mu}{r_e} - \frac{\mu}{a_d}}$$

Please note that these equations are also valid for the case of de-orbit from an initial circular orbit as described in the previous `cdeorbit.m` script.

The following is a typical user interaction with this script.

```
              program edeorbit

  < single impulse deorbit from elliptical orbits >


  please input the perigee altitude (kilometers)
  ? 285.798

  please input the apogee altitude (kilometers)
  ? 35785.922


  please input the entry altitude (kilometers)
  ? 111.252

  please input the entry flight path angle (degrees)
  ? -4
```

The following is the script output created for this example.

```
              program edeorbit

  < single impulse deorbit from elliptical orbits >

  initial orbit

  perigee altitude       285.798000  kilometers

  apogee altitude      35785.922000  kilometers

  semimajor axis       24414.000000  kilometers

  eccentricity             0.727044

  entry altitude         111.252000  kilometers

  entry fpa               -4.000000  degrees


  entry trajectory

  semimajor axis             24308.08290588  kilometers

  eccentricity                   0.73456961
```

```
perigee altitude              73.96381175  kilometers

apogee altitude            35785.92200000  kilometers

entry true anomaly           350.55084585  degrees

entry velocity             10317.40933180  meters/second

entry fpa                     -4.00000000  degrees

impulse-to-entry time        312.58844372  minutes

deorbit delta-v               22.29796787  meters/second
```

The software will also calculate and display the entry velocity and flight path angle relative to a rotating spherical Earth. The following is the relative flight path information for this example.

```
relative flight path coordinates

entry velocity              9845.40345708  meters/second

entry fpa                     -4.19210209  degrees
```

This MATLAB script will also graphically display the initial elliptic orbit and the de-orbit trajectory. The graphic display for this example is as follows where the red dots represent the original elliptical orbit and the blue dots represent the de-orbit trajectory, both at one minute intervals. The black circle is the surface of a spherical Earth and the distances are in Earth radii.