OSCILLATIONS FORCED BY F0\*SIN(\*t) OR F0\*COS(\*t) v1.02

This CAS-program finds for the differential equation (D. E.) of an oscillation given as

*1)* Mass \* d2y/dt2 + damp \* dy/dt + spring \* y = 0 (unforced)

*2)* Mass \* d2y/dt2 + damp \* dy/dt + spring \* y = Fcos\*cos(\*t) (forced)

*3)*  Mass \* d2y/dt2 + damp \* dy/dt + spring \* y = Fsin\*sin(\*t)

*4)* ….. = Fcos\*cos(\*t) + Fsin\*sin(\*t) ( combination of cases *2)* and *3)* )

the exact general solution and replaces the constants of integration C1, C2 by numerical values dependant to preset initial conditions t0, y0(t0), y'0(t0), which are named as tim ( = t0 ), loc ( = y0(t0), vel ( = y'0(t0) ).

The initial variables loc or vel must be <> 0 for case *1)* !!

The factors are in particular:

mass: mass *m* of system [kg], damp: damping factor *d* [kg/s] and spring: constant *c* of spring [N/m], Fcos, Fsin: external forces [N) and ( omega): frequency of the excitation [1/s] ( may be

0 ).

The equations *2)* and *3)* are customarily written as

d2y/dt2 + 2\*\*dy/dt + 2\*y = K0\*cos(\*t) or d2y/dt2 + 2\*\*dy/dt + 2\*y = K0\*sin(\*t),

where: 2\* damp/mass,2 spring/mass, K0  = F/mass

The program takes credit of the built-in desolve()-function, to find the numerical solution.

To start, press the Vars-key, then touch the CAS-field, if not highlighted and select ForcedOscill.

Complete the input line to: ForcedOscill(*mass,damp,spring, Fcos, Fsin,omega,tim,loc,vel*), then press Enter. The following calculation may take some seconds ( depending on the system ), then the results are displayed separated by the Enter-key: D. E. and initial condition (*fig.* *1*), solution y(t) with constants C1 and C2 and their numerical values of (*fig.* *2*), numerical solution of y(t) in exact- and approx-mode, frequency of the damped oscillation d = sqrt(2 -  ) s-1, T = 2\*/d s

f = 1/T Hz (*fig. 3*). If damp>0, the next screen shows Lehr´s damping ratio D = sqrt(2), where:

D < 0: System underdamped (weak damping), D = 0: Aperiodic borderline case,

D > 0: System overdamped,

and the steady state solution y(t),st = A\*sin(\*t +, which depicts the particular part of the solution caused by Fcos\*cos(\*t) or Fsin\*sin(\*t) (*fig. 4*). Now press Enter again to display the

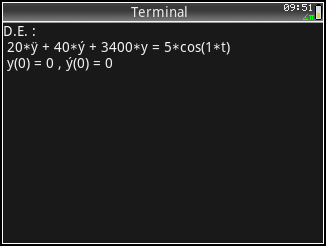
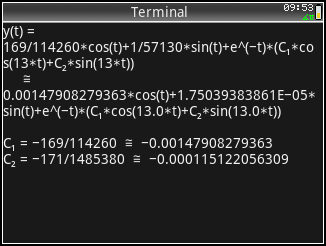
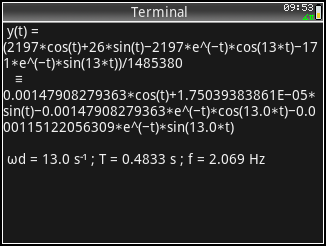
graph (*fig. 5*) depicting y(t) (green), y´(t) = v(t) (red), y(t),st (blue) and the difference y(t) - y(t),st

(brown line).

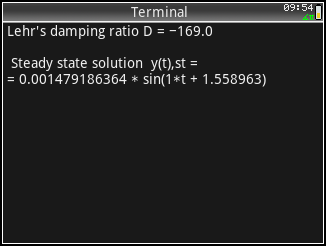
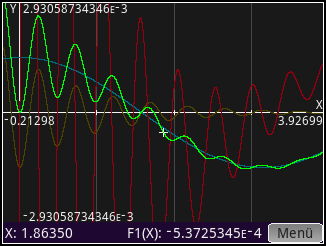
*EXAMPLE:*

A body with mass m = 20 kg and a spring factor c = 3400 N/m of the system is oscillating in a damping fluid with d = 40 kg/s, excited by a vibrating force F(t) = 5 N \* cos(t), ( 1 s-1Fsin = 0). Find the equation, the course of the oscillation. The movement starts at x0(=t0)=0 motionless from the origin (y(0)=y’(0)=0).

Start ForcedOscill as described above and key in the appropriate values: ForcedOscill(20,40,3400,5,0,1,0 0 0) Enter. The equation appropriate is shown ( *fig. 1* ), followed by the various parts of the solution *( fig. 2 – fig. 4* ). The graph ( *fig. 5* ) depicts the course of the oscillation: the damped part subsides after ~ 4 s and the excitation becomes dominant.

** Enter ** Enter 

*fig. 1 fig. 2 fig. 3*

*Enter * Enter **

*fig. 4 fig. 5*

*REFERENCES:*

“Mathematik für Ingenieure und Naturwissenschaftler“, L. Papula; F. Vieweg & Son Publishers, 1997

“Technische Mechanik”, J. Dankert/H. Dankert; Vieweg + Teubner publisher’s 2009

All parts of this program are copyrighted by Claus Dachselt. It is provided as a free release, for non-commercial purpose “as is” without warranty or responsibility of any kind. In no event will the copyright holder be liable for damages arising out of the use or inability to use the program.

*Claus Martin Dachselt*