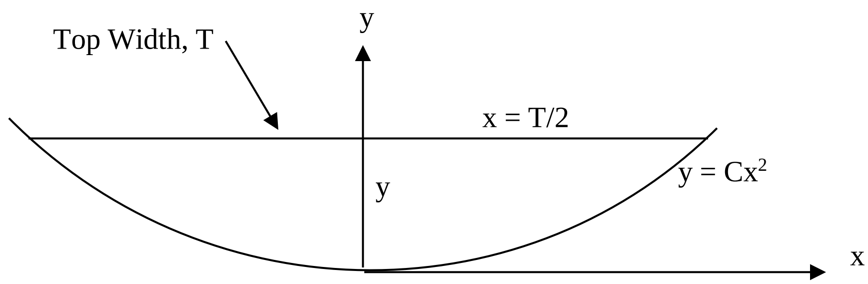
Uniform Flow in a Parabolic Channel

3/31/18 by ENS

This program calculates the normal depth of a parabolic channel in the form of $Y = CX^2$, where C is the X^2 coefficient also known as parabola "curvature". A known top width (T_1) and corresponding depth (Y_1) must be inputted to calculate the parabola curvature C. The wetted perimeter P is calculated using the exact formula per Chow as redefined by Merkley. See the enclosed attachment for the formulas and test problems used in writing the program.



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A. Parabolic Channel Equations from "Open Channel Hydraulics"

From the book "Open Channel Hydraulics" by Ven Te Chow, 1959, McGraw Hill, Table 2.1, "Generic Elements of Channel Sections", page 21, gives the following formulas for a parabolic channel.

Top Width $T = \frac{3A}{2y}$ (1)

Area $A = \frac{2}{3}Ty$ (2)

Approx. Hydraulic Radius $R^* = \frac{2T^2y}{3T^2+8y^2}$ (3)

Approx. Wetted Perimeter $P^* = T + \frac{8y^2}{3T}$ (4)

Exact Wetter Perimeter $P = \frac{T}{2}\left[\sqrt{1+x^2} + \frac{1}{x} * ln\left(x + \sqrt{1+x^2}\right)\right]$ (5)

where $x = \frac{4y}{T}$ (6)

*Satisfactory approximation for the interval 0 < x <= 1 where $x = \frac{4y}{T}$

B. Parabolic Channel Equations as a function C

Parabola equation $y = Cx^2$ (5)

Curvature C

From the above diagram, for a given depth y, x = T/2. Plugging T/2 into the parabola equation and solving for C gives the following equation.

Curvature $\mathrm{C} = \frac{4\mathrm{y}}{\mathrm{T}^2}$ (6)

C is the x^2 coefficient of the parabola equation and is referred to as the parabola curvature.

Top Width $\mathrm{T} = 2 * \sqrt{\frac{\mathrm{y}}{\mathrm{C}}}$ (7)

The above top width equation is derived from the Curvature C equation and solving for T.

Area A. For the area of the parabolic channel, we use Chow's area equation and replace T with the top width equation that specifies T in terms of y and C.

$\mathrm{A} = \frac{2}{3}\,\mathrm{T}\,\mathrm{y}$, replace T with $2 * \sqrt{\frac{\mathrm{y}}{\mathrm{C}}}$

$\mathrm{A} = \frac{2}{3} * 2 * \sqrt{\frac{\mathrm{y}}{\mathrm{C}}} * \mathrm{y}$, simplifying we get

$A = \frac{4}{3} * \frac{y^{1/2}}{C^{1/2}} * y$, simplifying we get

$$A = \frac{4y^{3/2}}{3C^{1/2}} \tag{8}$$

Exact Wetted Perimeter P

Convert Chow's exact wetted perimeter formula to be a function of C

Recall the Wetted Perimeter equation from Chow,

$$P\ (\text{Chow}) = \frac{T}{2} * \left[\sqrt{1 + x^2} + \frac{1}{x} * \ln\left(x + \sqrt{1 + x^2}\right)\right]$$

We replace the x term with $\frac{4y}{T}$, which is the definition of x listed in the footnote of Table 2.1.

$$P(\text{Chow}) = \frac{T}{2} * \left[\sqrt{1 + \left(\frac{4y}{T}\right)^2} + \frac{1}{\left(\frac{4y}{T}\right)} * \ln\left(\frac{4y}{T} + \sqrt{1 + \left(\frac{4y}{T}\right)^2}\right)\right]$$

Simplifying the middle term we get

$$P(\text{Chow}) = \frac{T}{2} * \left[\sqrt{1 + \left(\frac{4y}{T}\right)^2} + \frac{T}{4y} * \ln\left(\frac{4y}{T} + \sqrt{1 + \left(\frac{4y}{T}\right)^2}\right)\right]$$

Substitute T using the equation $T = 2\sqrt{\frac{y}{C}}$ we get

$$P(\text{Chow}) = \frac{2\sqrt{\frac{y}{C}}}{2} * \left[\sqrt{1 + \left(\frac{4y}{2\sqrt{\frac{y}{C}}}\right)^2} + \frac{2\sqrt{\frac{y}{C}}}{4y} * \ln\left(\frac{4y}{2\sqrt{\frac{y}{C}}} + \sqrt{1 + \left(\frac{4y}{2\sqrt{\frac{y}{C}}}\right)^2}\right)\right]$$

Simplifying we get

$$P(\text{Chow}) = \sqrt{\frac{y}{C}} * \left[\sqrt{1 + \left(\frac{2y}{\sqrt{\frac{y}{C}}}\right)^2} + \frac{\sqrt{\frac{y}{C}}}{2y} * \ln\left(\frac{2y}{\sqrt{\frac{y}{C}}} + \sqrt{1 + \left(\frac{2y}{\sqrt{\frac{y}{C}}}\right)^2}\right)\right] \tag{9}$$

This equation can be simplified further, but this current format is sufficient for the program. It shows that Chow's wetted perimeter formula can be expressed as a function of y and C. Chow's translated equation is very similar to the following Merkley's equation for P.

Exact Wetted Perimeter Formula from Merkley

$$P(\text{Merkley}) = 2C\sqrt{\frac{y}{C}\left(\frac{y}{C} + \frac{1}{4C^2}\right)} + \frac{1}{2C}\ \ln\left[2C\left(\sqrt{\frac{y}{C}} + \sqrt{\frac{y}{C} + \frac{1}{4C^2}}\right)\right] \tag{10}$$

This formula for the wetted perimeter was found on the internet from a 2004 lecture slide by Gary P Merkley, Utah State University, where he solved the integral of the wetted perimeter of the function $y=Cx^2$. He solved it for ½ of the parabola, the equation above is for the full parabola.

Hydraulic Mean Depth $D_h = \frac{A}{T}$ (13)

Froude Number $Fr = \frac{V}{\sqrt{gD_h}}$ (14)

where $g = 32.2\frac{ft}{s^2} = 9.81\ \frac{m}{s^2}$

C. Manning's Equation for the successive approximation method to solve for depth.

Hydraulic Radius $R = \frac{A}{P}$

$Q = \frac{k}{n} A R^{2/3} S^{1/2}$, replace R with A/P and simplify

$Q = \frac{k}{n} A \left(\frac{A}{P}\right)^{2/3} S^{1/2} = \frac{k}{n} A \frac{A^{2/3}}{P^{2/3}} S^{1/2} = \frac{k}{n} \frac{A^{5/3}}{P^{2/3}} S^{1/2}$, replace A with $\frac{4}{3}\frac{y^{3/2}}{C^{1/2}}$ and simplify

$Q = \frac{k}{n} \frac{\left(\frac{4}{3}\frac{y^{3/2}}{C^{1/2}}\right)^{5/3}}{P^{2/3}} S^{1/2}$, solve for y

$\left(\frac{4}{3}\frac{y^{3/2}}{C^{1/2}}\right)^{5/3} = \frac{QnP^{2/3}}{k*S^{1/2}}$, take the 3/5 power of each side and simplify

$\frac{4}{3}\frac{y^{3/2}}{C^{1/2}} = \left(\frac{QnP^{2/3}}{k*S^{1/2}}\right)^{3/5} = \frac{Q^{3/5}n^{3/5}\left(P^{2/3}\right)^{3/5}}{k^{3/5}*\left(S^{1/2}\right)^{3/5}} = \frac{Q^{3/5}n^{3/5}P^{6/15}}{k^{3/5}*S^{3/10}}$, isolate the y term

$y^{3/2} = \frac{3C^{1/2}}{4}\left(\frac{QnP^{2/3}}{k*S^{1/2}}\right)^{3/5}$, take the $\frac{2}{3}$ power on each side of the equation

$y_{new} = \left(\frac{3C^{1/2}}{4}\right)^{2/3}\left(\frac{QnP^{2/3}}{k*S^{1/2}}\right)^{6/15}$ (15)

The successive approximations procedure for a parabolic channel was very briefly discussed in Hydrologic and Hydraulic Computations on Small Programmable Calculators by Thomas E Crowley II, Iowa Institute of Hydraulic Research, 1977, page 512 and 526. The area and wetted perimeter are both functions of y and C. The area variable in Manning's equation is replaced with its function f(y,c), but the P variable is left alone. Manning's equation is rearranged to solved for y, where y = f(C, Q, n, S, P). The wetted perimeter equation P = f(y,c) is used separately from Manning's equation. For the successive approximations procedure, you assume an initial guess for y. Solve P = f(y,c) using the initial guess for y. Using this P, you then solve Manning's Eq: y_{new} = f(C, Q, n, S, P). This will give you a more precise y. You use the more precise y and repeat the process giving you successively more accurate approximations of y. The successive approximations procedure is generally covered in a little more detail in the Handbook of Hydraulics, 7^{th} ed., by Brator, King, Lindell and Wei, McGraw Hill, 1996, page 13.4.

D. Determining a more precise Manning's k value

From wikipedia.org, Manning's n has units of n $\frac{s}{m^{1/3}}$ and Manning's k is a conversion factor to convert the metric n-value to English units of n $\frac{s}{ft^{1/3}}$ To convert n from metric to English units we have

$$n\ \frac{m^{1/3}}{s}\frac{\frac{(3.28083989501\ ft)^{1/3}}{s}}{1\frac{m^{1/3}}{s}} = n\frac{m^{1/3}}{s} * \frac{1.4859185775\frac{ft^{1/3}}{s}}{1\frac{m^{1/3}}{s}}$$

so, k = 1.4859185775

E. Test Problem 1 - Solve for Q given S,y,n,T1,Y1

A grassy parabolic channel at a slope of 0.008 has water flowing at a depth of 2 ft. The top width of the water is 20 ft wide. Assume n = 0.05. Solve for the flow rate Q.

1. Using Chow's Equations

Area $A = \frac{2}{3}Ty = \frac{2}{3}(20)(2) = 26.6667$

$$x = \frac{4y}{T} = \frac{4(2)}{20} = 0.4$$

Exact Wetted Perimeter $P\ (Chow) = \frac{T}{2}\left[\sqrt{1+x^2} + \frac{1}{x} * ln\left(x + \sqrt{1+x^2}\right)\right]$

$$P = \frac{20}{2}\left[\sqrt{1+(0.4)^2} + \frac{1}{0.4} * ln\left(0.4 + \sqrt{1+(0.4)^2}\right)\right] = 20.5212$$

Hydraulic Radius $R = \frac{A}{P} = \frac{26.6667}{20.5212} = 1.2995$ ft

Flow Rate $Q = \frac{k}{n}AR^{2/3}S^{1/2} = \frac{1.4859185775}{0.05}(26.6667)(1.2995)^{2/3}(0.008)^{1/2} = 84.4093$

Top Width $T = \frac{3A}{2y} = \frac{3(26.6667)}{2(2)} = 20.0000$

Velocity $V = \frac{Q}{A} = \frac{84.4093}{26.6667} = 3.1653$ ft/s

Hydraulic Mean Depth $D_h = \frac{A}{T} = \frac{26.6667}{20} = 1.3333$ ft

Froude Number $Fr = \frac{V}{\sqrt{gD_h}} = \frac{3.1653}{\sqrt{32.2 * 1.3333}} = 0.4831$

2. Using Equations expressed as a function of C

Curvature $C = \frac{4y}{T^2} = \frac{4(2)}{(20)^2} = 0.02$

Area $A = \frac{4}{3}\frac{y^{3/2}}{C^{1/2}} = \frac{4}{3}\frac{(2)^{3/2}}{(0.02)^{1/2}} = 26.6667$ sf

Wetted Perimeter P using Chow's equation converted to be a function of C

$$P\ (\text{Chow conv.}) = \sqrt{\frac{y}{C}} * \left[\sqrt{1 + \left(\frac{2y}{\sqrt{\frac{y}{C}}}\right)^2} + \frac{\sqrt{\frac{y}{C}}}{2y} * \ln\left(\frac{2y}{\sqrt{\frac{y}{C}}} + \sqrt{1 + \left(\frac{2y}{\sqrt{\frac{y}{C}}}\right)^2}\right)\right]$$

$$P\ (\text{Chow conv.}) = \sqrt{\frac{2}{0.02}} * \left[\sqrt{1 + \left(\frac{2*2}{\sqrt{\frac{2}{0.02}}}\right)^2} + \frac{\sqrt{\frac{2}{0.02}}}{2*2} * \ln\left(\frac{2*2}{\sqrt{\frac{2}{0.02}}} + \sqrt{1 + \left(\frac{2*2}{\sqrt{\frac{2}{0.02}}}\right)^2}\right)\right]$$

$P\ (\text{Chow conv.}) = 20.5212$ ft

Wetted Perimeter $P\ (\text{Merkley}) = 2C\sqrt{\frac{y}{C}\left(\frac{y}{C} + \frac{1}{4C^2}\right)} + \frac{1}{2C}\ \ln\left[2C\left(\sqrt{\frac{y}{C}} + \sqrt{\frac{y}{C} + \frac{1}{4C^2}}\right)\right]$

$$P = 2(0.02)\sqrt{\frac{2}{0.02}\left(\frac{2}{0.02} + \frac{1}{4(0.02)^2}\right)} + \frac{1}{2(0.02)}\ \ln\left[2(0.02)\left(\sqrt{\frac{2}{0.02}} + \sqrt{\frac{2}{0.02} + \frac{1}{4(0.02)^2}}\right)\right]$$

$P = 20.5212$ ft

The converted Chow equation for the wetted perimeter gives the same answer as Merkley's equations. Use Merkley's equation in the program, because it is more concise.

Hydraulic Radius $R = \frac{A}{P} = \frac{26.6667}{20.5212} = 1.2995$

Flow Rate $Q = \frac{k}{n}AR^{2/3}S^{1/2} = \frac{1.4859185775}{0.05}(26.6667)(1.2995)^{2/3}(0.008)^{1/2}$=84.4093

Top Width $T = 2 * \sqrt{\frac{y}{C}} = 2 * \sqrt{\frac{2}{0.02}} = 20$ ft

Velocity $V = \frac{Q}{A} = \frac{84.4093}{26.6667} = 3.1653$ ft/s

Hydraulic Mean Depth $D_h = \frac{A}{T} = \frac{26.6667}{20} = 1.3333$ ft The hydraulic mean depth is used to calculate the Froude Number.

Froude Number $Fr = \frac{V}{\sqrt{gD_h}} = \frac{3.1653}{\sqrt{32.2*1.3333}} = 0.4831$

F. Test Problem 2 – Solve y given S,Q,n,T1,Y1 by the successive approx. method

In the previous problem, the water surface width was 20 ft and the depth was 2 ft. The channel had a curvature C of 0.02. What is the surface width, if the water depth was 2.5 feet?

Top Width $T = 2 * \sqrt{\frac{y}{C}} = 2 * \sqrt{\frac{2.5}{0.02}} = 22.36068$. Rewriting the problem statement to include the channel geometry we get:

A grassy swale at a slope of 0.008 has water flowing at a rate of 84.4093 cfs. The swale has a parabolic shape and is 2.5 ft deep by 22.36068 ft wide. Assume n = 0.05. Solve for the depth of flow in the swale.

Curvature $C = \frac{4y}{T^2} = \frac{4(2.5)}{(22.36068)^2} = 0.02$

Assume an initial depth of 0.1 ft, and solve for P

$$P = 2C\sqrt{\frac{y}{C}\left(\frac{y}{C} + \frac{1}{4C^2}\right)} + \frac{1}{2C}\ln\left[2C\left(\sqrt{\frac{y}{C}} + \sqrt{\frac{y}{C} + \frac{1}{4C^2}}\right)\right]$$

$$P = 2(0.02)\sqrt{\frac{0.1}{0.02}\left(\frac{0.1}{0.02} + \frac{1}{4(0.02)^2}\right)} + \frac{1}{2(0.02)}\ln\left[2(0.02)\left(\sqrt{\frac{0.1}{0.02}} + \sqrt{\frac{0.1}{0.02} + \frac{1}{4(0.02)^2}}\right)\right] = 4.4781$$

Using P, solve for the revised depth

$$y_{new} = \left(\frac{3C^{1/2}}{4}\right)^{2/3}\left(\frac{QnP^{2/3}}{k * S^{1/2}}\right)^{6/15}$$

$$y_{new} = \left(\frac{3(0.02)^{1/2}}{4}\right)^{2/3}\left(\frac{84.4093(0.05)[4.4781]^{2/3}}{1.4859185775 * (0.008)^{1/2}}\right)^{6/15} = 1.3327$$

Using the revised depth repeat the procedure for successive new depths.

Depth Y	Wetted Perimeter P	Revised Depth Ynew	Error = abs(Y-Ynew)
0.1	4.4781	1.3327	1.2327
1.3327	16.6117	1.8904	0.5577
1.8904	19.9238	1.9843	0.0939
1.9843	20.4365	1.9978	0.0135
1.9978	20.5094	1.9997	0.0019
1.9997	20.5196	2.0000	0.0003
2.0000	20.5212	2.0000	0.0000

Y = Ynew = 2.0000

$$\text{Top Width T} = 2 * \sqrt{\frac{y}{C}} = 2 * \sqrt{\frac{2.0000}{0.02}} = 20.0000$$

$$\text{Area A} = \frac{4}{3}\frac{y^{3/2}}{C^{1/2}} = \frac{4}{3}\frac{(2.0000)^{3/2}}{(0.02)^{1/2}} = 26.6667 \text{ sf}$$

$$\text{Velocity V} = \frac{Q}{A} = \frac{84.4093}{26.6667} = 3.1653 \text{ ft/s}$$

$$\text{Hydraulic Mean Depth } D_h = \frac{A}{T} = \frac{26.6667}{20} = 1.3333 \text{ ft}$$

$$\text{Froude Number Fr} = \frac{V}{\sqrt{gD_h}} = \frac{3.1653}{\sqrt{32.2 * 1.3333}} = 0.4831$$

G. Test Problem 3 – Solve for S given Q, y, n, T1, Y1

A grassy swale has water flowing at a rate of 84.4093 cfs and a depth of 2 ft. The swale has a parabolic shape and is 2.5 ft deep and 22.36068 ft wide. Assume n = 0.05. Solve for the slope of the swale.

Or

A grassy swale has water flowing at a rate of 84.4093 cfs. The water depth is 2 ft and a surface width is 20 ft. The swale has a parabolic shape. Assume n = 0.05. Solve for the slope of the swale.

$$\text{Curvature C} = \frac{4y}{T^2} = \frac{4(2.5)}{(22.36068)^2} = 0.02 \quad \text{or} \quad C = \frac{4y}{T^2} = \frac{4(2)}{(2)^2} = 0.02$$

$$\text{Area A} = \frac{4}{3}\frac{y^{3/2}}{C^{1/2}} = \frac{4(2)^{3/2}}{3(0.02)^{1/2}} = 26.6667 \text{ sf}$$

$$\text{Wetted Perimeter P} = 2C\sqrt{\frac{y}{C}\left(\frac{y}{C} + \frac{1}{4C^2}\right)} + \frac{1}{2C}\ln\left[2C\left(\sqrt{\frac{y}{C}} + \sqrt{\frac{y}{C} + \frac{1}{4C^2}}\right)\right]$$

$$P = 2(0.02)\sqrt{\frac{2}{0.02}\left(\frac{2}{0.02} + \frac{1}{4(0.02)^2}\right)} + \frac{1}{2(0.02)}\ln\left[2(0.02)\left(\sqrt{\frac{2}{0.02}} + \sqrt{\frac{2}{0.02} + \frac{1}{4(0.02)^2}}\right)\right]$$

$$P = 20.5212 \text{ ft}$$

$$\text{Hydraulic Radius R} = \frac{A}{P} = \frac{26.6667}{20.5212} = 1.2995 \text{ ft}$$

Given $Q = \frac{k}{n}AR^{2/3}S^{1/2}$, solve for S

Slope $S^{1/2} = \frac{nQ}{kAR^{2/3}}$, square both side

$$\text{Slope } S = \left(\frac{nQ}{kAR^{2/3}}\right)^2 = \left(\frac{0.05*84.4093}{1.4859185775(26.6667)(1.2995)^{2/3}}\right)^2 = 0.0080$$

$$\text{Top Width } T = 2*\sqrt{\frac{y}{C}} = 2*\sqrt{\frac{2}{0.02}} = 20.0000$$

$$\text{Velocity } V = \frac{Q}{A} = \frac{84.4093}{26.6667} = 3.1653 \text{ ft/s}$$

$$\text{Hydraulic Mean Depth } D_h = \frac{A}{T} = \frac{26.6667}{20} = 1.3333 \text{ ft}$$

$$\text{Froude Number } Fr = \frac{V}{\sqrt{gD_h}} = \frac{3.1653}{\sqrt{32.2*1.3333}} = 0.4831$$

H. Test Problem 4 – Solve for n given Q, y, S, T1, Y1

A grassy swale has water flowing at a rate of 84.4093 cfs and a depth of 2 ft. The swale has a parabolic shape and is 2.5 ft deep and 22.36068 ft wide. The swale slope is 0.008. Solve for Manning's n.

Or

A grassy swale has water flowing at a rate of 84.4093 cfs and a depth of 2 ft and a surface width of 20 ft. The swale has a parabolic shape. The swale slope is 0.008. Solve for Manning's n.

$$\text{Curvature } C = \frac{4y}{T^2} = \frac{4(2.5)}{(22.36068)^2} = 0.02 \quad \text{or} \quad C = \frac{4y}{T^2} = \frac{4(2)}{(20)^2} = 0.02$$

$$\text{Area } A = \frac{4}{3}\frac{y^{3/2}}{C^{1/2}} = \frac{4*(2)^{3/2}}{3*(0.02)^{1/2}} = 26.6667\text{sf}$$

$$\text{Wetted Perimeter } P = 2C\sqrt{\frac{y}{C}\left(\frac{y}{C}+\frac{1}{4C^2}\right)} + \frac{1}{2C}\ln\left[2C\left(\sqrt{\frac{y}{C}}+\sqrt{\frac{y}{C}+\frac{1}{4C^2}}\right)\right]$$

$$P = 2(0.02)\sqrt{\frac{2}{0.02}\left(\frac{2}{0.02}+\frac{1}{4(0.02)^2}\right)} + \frac{1}{2(0.02)}\ln\left[2(0.02)\left(\sqrt{\frac{2}{0.02}}+\sqrt{\frac{2}{0.02}+\frac{1}{4(0.02)^2}}\right)\right]$$

$$P = 20.5212 \text{ ft}$$

$$\text{Hydraulic Radius } R = \frac{A}{P} = \frac{26.6667}{20.5212} = 1.2995 \text{ ft}$$

$$\text{Flow Rate } Q = \frac{k}{n}AR^{2/3}S^{1/2} \quad \text{, solve for n}$$

$$\text{n value} = \frac{k}{Q}AR^{2/3}S^{1/2} = \frac{1.4859185775}{84.4090}26.6667(1.2995)^{2/3}(0.008)^{1/2} = 0.05$$

$\text{Top Width } T = 2 * \sqrt{\frac{y}{C}} = 2 * \sqrt{\frac{2}{0.02}} = 20.0000$

$\text{Velocity } V = \frac{Q}{A} = \frac{84.4093}{26.6667} = 3.1653 \text{ ft/s}$

$\text{Hydraulic Mean Depth } D_h = \frac{A}{T} = \frac{26.6667}{20} = 1.3333 \text{ ft}$

$\text{Froude Number } Fr = \frac{V}{\sqrt{gD_h}} = \frac{3.1653}{\sqrt{32.2*1.3333}} = 0.4831$

I. Program Variables (Preliminary)

Input Variables

T1	known top width for a given depth	ft	m
Y1	known depth	ft	m
Q	Flow Rate	ft^3/s	m^3/s
N	Manning n value	$\frac{s}{m^{1/3}}$	$\frac{s}{m^{1/3}}$
S	Slope in decimal format	ft/ft	m/m
Y	Depth of flow	ft	m
UnitsP	flag to specify English or metric units	1	2
SolveNum	flag for the variable to solve for:		
	1 Solve Y, 2 Solve Q, 3 Solve S, 4 Solve N		

Program Variables

K	Manning's k value	1.4859185775	1
C	Parabola Curvature		
T	Top Width	ft	m
A	Area	ft^2	m^2
P	Wetted Perimeter		
R	Hydraulic Radius		
Ynew	revised depth Y for successive approx. method		
Yerror	Error between depth Y and new depth Y		
V	Velocity		
G	accel const	$32.2 \frac{ft}{s^2}$	$9.81 \frac{m}{s^2}$
Fr	Froude Number		

J. Program Outline (Preliminary)

Input

Case 1 English Units

$G = 32.2 \frac{ft}{s^2}$

K = 1.4859185775

Case 2 Metric Units

$G = 9.81\ \frac{m}{s^2}$

K = 1

Case 1 "Solve for Q given S, Y, N, T1, Y1"

Solve C using subroutine "calcC" given Y1, T1

Solve A using subroutine "calcA" given Y, C

Solve P using subroutine "calcP" given Y, C

Solve R using subroutine "calcR" given A, P

$Q = \frac{k}{n} AR^{2/3} S^{1/2}$ given K, N, A, R, S

Solve V using subroutine "calcV" given Q, A

Solve T using subroutine "calcT" given Y, C

Solve Dh using subroutine "calcDh" given A, T

Solve Fr using subroutine "calcFr" given G, V, Dh

Print output using subroutine "prntout"

Case 2 "Solve for Y given S, Q, N, T1, Y1"

Solve C using subroutine "calcC" given Y1, T1

Initial depth Y = 0.1 (ft, m)

Loop

Solve P using subroutine "calcP" given Y, C

$y_{new} = \left(\frac{3C^{1/2}}{4}\right)^{2/3} \left(\frac{QnP^{2/3}}{k * S^{1/2}}\right)^{6/15}$ given K, C, Q, N, P, S

Error = Ynew – Y

Is error < 0.000001 then exit loop

Y = Ynew

End loop

Using the final Y, solve for P using sub "calcP" given Y, C

Solve A using subroutine "calcA" given Y, C

Solve V using subroutine "calcV" given Q, A

Solve T using subroutine "calcT" given Y, C

Solve Dh using subroutine "calcDh" given A, T

Solve Fr using subroutine "calcFr" given G, V, Dh

Print output using subroutine "prntout"

Case 3 "Solve for S given Q, Y, N, T1, Y1"

Solve C using subroutine "calcC" given Y1, T1

Solve A using subroutine "calcA" given Y, C

Solve P using subroutine "calcP" given Y, C

Solve R using subroutine "calcR" given A, P

$S = \left(\frac{nQ}{kAR^{2/3}}\right)^2$ given K, N, Q, A, R

Solve V using subroutine "calcV" given Q, A

Solve T using subroutine "calcT" given Y, C

Solve Dh using subroutine "calcDh" given A, T

Solve Fr using subroutine "calcFr" given G, V, Dh

Print output using subroutine "prntout"

Case 4 "Solve for N given Q, Y, S, T1, Y1"

Solve C using subroutine "calcC" given Y1, T1

Solve A using subroutine "calcA" given Y, C

Solve P using subroutine "calcP" given Y, C

Solve R using subroutine "calcR" given A, P

$$n = \frac{k}{Q} A R^{2/3} S^{1/2}$$

Solve V using subroutine "calcV" given Q, A

Solve T using subroutine "calcT" given Y, C

Solve Dh using subroutine "calcDh" given A, T

Solve Fr using subroutine "calcFr" given G, V, Dh

Print output using subroutine "prntout"

Subroutine "calcC", solve C given Y1, T1

$$C = \frac{4y}{T^2} = \frac{4Y1}{T1^2}$$

Subroutine "calcA", solve A given Y, C

$$A = \frac{4}{3} \frac{y^{3/2}}{C^{1/2}}$$

Subroutine "calcP", solve P given Y, C

$$P = 2C\sqrt{\frac{y}{C}\left(\frac{y}{C} + \frac{1}{4C^2}\right)} + \frac{1}{2C} \ln\left[2C\left(\sqrt{\frac{y}{C}} + \sqrt{\frac{y}{C} + \frac{1}{4C^2}}\right)\right]$$

Subroutine "calcR", solve R given A, P

$$R = \frac{A}{P}$$

Subrouting "calcV", solve V given Q, A

$$V = \frac{Q}{A}$$

Subrouting "calcT", solve T given Y, C

$$T = 2 * \sqrt{\frac{y}{C}}$$

Subrouting "calcDh", solve Dh given A, T

$$D_h = \frac{A}{T}$$

Subrouting "calcFr", solve Fr given G, V, Dh

$$Fr = \frac{V}{\sqrt{g D_h}}$$

Subroutine "prntout"

Parabolic Channel

C, Y1, T1 =

N =

Q =

S =

Y =

V =

T =

A =

R =

Dh =
Fr =

K. Comparison of Crowley's program with this program using Crowley's examples

In Hydrologic and Hydraulic Computations on Small Programmable Calculators by Thomas E Crowley II, Iowa Institute of Hydraulic Research, 1977, page 526, he gives three examples of his program that calculated the depth of flow for a parabolic channel. In each of the examples, the parabolic channel shape was defined by having a top width of 10 ft at a depth of 1 ft, Manning n = 0.030, and error = 0.001. Crowley's program calculated the following depth and areas for the three examples.

Crowley's Examples

Q cfs	100	200	300
S	0.009028	0.009028	0.016
Depth ft	1.96106	2.71469	2.87148
Area sf	18.308	29.819	32.439
k	1.49	1.49	1.49

Using this program, the depth and area was calculated for Crowley's three examples.

Depth and area calculated by this program

Q cfs	100	200	300
S	0.009028	0.009028	0.016
Depth ft	1.9637	2.7183	2.8753
Area sf	18.3447	29.879	32.5046
k	1.4859185775	1.4859185775	1.4859185775

The percent difference between the results from this program and Crowley's

Depth ft	0.13%	0.13%	0.13%
Area sf	0.20%	0.20%	0.20%
k	-0.27%	-0.27%	-0.27%

The difference in the results between this program and Crowley's program is probably due to the difference in the k values.

L. Test Problem 5 - Solve for Q given S,y,n,T1,Y1 using Metric units

A grassy parabolic channel at a slope of 0.008 has water flowing at a depth of 0.6 meters. The top width of the water is 6 meters wide. Assume n = 0.05. Solve for the flow rate Q.

1. Using Chow's Equations

Area $A = \frac{2}{3}Ty = \frac{2}{3}(6)(0.6) = 2.4\ \text{m}^2$

$$x = \frac{4y}{T} = \frac{4(0.6)}{6} = 0.4$$

Exact Wetted Perimeter $P\ (Chow) = \frac{T}{2}\left[\sqrt{1+x^2} + \frac{1}{x} * ln\left(x + \sqrt{1+x^2}\right)\right]$

$$P = \frac{6}{2}\left[\sqrt{1+(0.4)^2} + \frac{1}{0.4} * ln\left(0.4 + \sqrt{1+(0.4)^2}\right)\right] = 6.1563 \text{ m}$$

Hydraulic Radius $\mathrm{R} = \frac{\mathrm{A}}{\mathrm{P}} = \frac{2.4}{6.1563} = 0.3898$ m

Flow Rate $\mathrm{Q} = \frac{\mathrm{k}}{\mathrm{n}}\mathrm{AR}^{2/3}\mathrm{S}^{1/2} = \frac{1}{0.05}(2.4)(0.3898)^{2/3}(0.008)^{1/2} = 2.2909\ m^3/s$

Top Width $T = \frac{3A}{2y} = \frac{3(2.4)}{2(0.6)} = 6$ m

Velocity $\mathrm{V} = \frac{\mathrm{Q}}{\mathrm{A}} = \frac{2.2909}{2.4} = 0.9545$ m/s

Hydraulic Mean Depth $\mathrm{D_h} = \frac{\mathrm{A}}{\mathrm{T}} = \frac{2.4}{6} = 0.4$ m

Froude Number $\mathrm{Fr} = \frac{\mathrm{V}}{\sqrt{\mathrm{g}\mathrm{D_h}}} = \frac{0.9545}{\sqrt{9.81 * 0.4}} = 0.4818$

2. <u>Using Equations expressed as a function of C</u>

Curvature $\mathrm{C} = \frac{4\mathrm{y}}{\mathrm{T}^2} = \frac{4(0.6)}{(6)^2} = 0.06667$

Area $\mathrm{A} = \frac{4}{3}\frac{\mathrm{y}^{3/2}}{\mathrm{C}^{1/2}} = \frac{4}{3}\frac{(0.6)^{3/2}}{(0.06667)^{1/2}} = 2.3999\ m^2$

Wetted Perimeter P (Merkley) $= 2\mathrm{C}\sqrt{\frac{\mathrm{y}}{\mathrm{C}}\left(\frac{\mathrm{y}}{\mathrm{C}} + \frac{1}{4\mathrm{C}^2}\right)} + \frac{1}{2\mathrm{C}}\ \ln\left[2\mathrm{C}\left(\sqrt{\frac{\mathrm{y}}{\mathrm{C}}} + \sqrt{\frac{\mathrm{y}}{\mathrm{C}} + \frac{1}{4\mathrm{C}^2}}\right)\right]$

$$\mathrm{P} = 2(0.06667)\sqrt{\frac{0.6}{0.06667}\left(\frac{0.6}{0.06667} + \frac{1}{4(0.06667)^2}\right)} + \frac{1}{2(0.06667)}\ \ln\left[2(0.06667)\left(\sqrt{\frac{0.6}{0.06667}} + \sqrt{\frac{0.6}{0.06667} + \frac{1}{4(0.06667)^2}}\right)\right]$$

$$\mathrm{P} = 6.1562\ m$$

Hydraulic Radius $\mathrm{R} = \frac{\mathrm{A}}{\mathrm{P}} = \frac{2.3999}{6.1562} = 0.3898$ m

Flow Rate $\mathrm{Q} = \frac{\mathrm{k}}{\mathrm{n}}\mathrm{AR}^{2/3}\mathrm{S}^{1/2} = \frac{1}{0.05}(2.3999)(0.3898)^{2/3}(0.008)^{1/2} = 2.2908\ \mathrm{m^3/s}$

Top Width $\mathrm{T} = 2 * \sqrt{\frac{\mathrm{y}}{\mathrm{C}}} = 2 * \sqrt{\frac{0.6}{0.06667}} = 5.9998$ m ft

Velocity $\mathrm{V} = \frac{\mathrm{Q}}{\mathrm{A}} = \frac{2.2908}{2.3999} = 0.9545\ m/s$

Hydraulic Mean Depth $D_h = \frac{A}{T} = \frac{2.3999}{6} = 0.4$ m The hydraulic mean depth is used to calculate the Froude Number.

Froude Number $Fr = \frac{V}{\sqrt{gD_h}} = \frac{0.9545}{\sqrt{9.81 * 0.4}} = 0.4818$

M. Program Listing

```
//Uniform Flow in a Parabolic Channel
// 3/31/2018 by ENS

//Variable Declarations
EXPORT Y1,T1,Dh,Fr,Ynew,Error;
EXPORT UnitsP,CFScms,FTm,FPSmps,SFsm; //Units case number and unit labels for printing
Export SolveNum,SolvedQ,SolvedY,SolvedS,SolvedN; //"solve for" case number and solved variable
flag for printing

//Subroutine Declarations
CalcC();
CalcA();
CalcP();
CalcR();
CalcV();
CalcT();
CalcDh();
CalcFr();

//Main Program
EXPORT ParabolicChan()
BEGIN

//Initialize the solved variable flag to be all blank
//These variables are used to flag the variable that was solved for when printing the output
SolvedQ:="";
SolvedY:="";
SolvedS:="";
SolvedN:="";

//Retrive the old values of depth Y1, top width T1, UnitsP, and SolveNum that was stored
//in the system variables from the previous running of the program so that they are displayed
//on the input screen
Y1:=L;
T1:=M;
UnitsP:=E;
SolveNum:=F;

INPUT(
{
{UnitsP,{"English Units","Metric Units"},{60,40,0}},
{Z,1,{95,5,1}},
{T1,[0],{28,20,2}},
{Y1,[0],{80,20,2}},
{Z,1,{95,5,3}},
{Q,[0],{28,20,4}},
{Y,[0],{80,20,4}},
{S,[0],{28,20,5}},
{N,[0],{80,20,5}},
{SolveNum,{"Solve for flowrate Q","Solve for depth Y", "Solve for slope S","Solve for Manning's
N"},{45,55,6}}
},
"Parabolic Channel Flow y=Cx^2",
{
" ",
"Enter channel geometry (or any known T1,Y1)",
"width T1 =", "depth Y1 =",
"Enter any 3 variables and solve for the 4th ",
"flowrate Q =", "depth Y =",
```

```
"slope S =", "Manning's N =",
" "
},
{
"Select English or Metric units",
"This check box is not used.",
"Enter the channel top width T1 (ft, m)",
"Enter the channel depth Y1 (ft, m)",
"This check box is not used.",
"Enter flowrate Q (cfs, m^3/s)",
"Enter depth Y (ft, m)",
"Enter slope in decimal format (ft/ft, m/m)",
"Enter Manning's N value",
"Select which variable to solve for."
});
// Input format that was used
// {variable,[0],{left%,length%,row number}, where [0] is for real numbers, left% is for the
right edge of the input box
// {variable,1,{left%,length%,row number}, where 1 is for a checkbox
// {variable,{"Text1,"Text2"},{left%,length%,row number}, format for a dropbox aka choose box
// The checkboxes were used just to show text on the input screeen

//English and Metric Unit definitions
CASE
//Case 1 English Units
IF UnitsP=1 THEN
K:=1.4859185775; //manning's k value
G:=32.2; //gravitational constant
//Unit labels for printing the results
CFScms:=" cfs";
FTm:=" ft";
FPSmps:=" ft/s";
SFsm:=" ft^2";
END;
//Case 2 Metric Units
IF UnitsP=2 THEN
K:=1; //manning's k value
G:=9.81; //gravitational constant
//Unit labels for printing the results
CFScms:=" m^3/s";
FTm:=" m";
FPSmps:=" m/s";
SFsm:=" m^2";
END;
END; // end case

CASE
//Case 1 Solve flowrate Q given S,Y,N,T1,Y1
IF SolveNum=1 THEN
CalcC();
CalcA();
CalcP();
CalcR();
Q:=(K/N)*A*R^(2/3)*S^(1/2);
CalcV();
CalcT();
CalcDh();
CalcFr();
SolvedQ:="** "; //flag Q when printing
END;

//Case 2 Solve depth Y given S,Q,N,T1,Y1
IF SolveNum=2 THEN
CalcC();
Ynew:=0;
Y:=0.1; //assumed initial depth (ft,m)
//Solve depth Y by successive approximation method
For I FROM 1 TO 50 DO
CalcP();
Ynew:=(3*C^(1/2)/4)^(2/3)*(Q*N*P^(2/3)/(K*S^(1/2)))^(6/15);
Error:=ABS(Ynew-Y);
```

```
If Error<0.0000001 then
Y:=Ynew; //save the final Y value
CalcP(); //recalculate P using the final Y value
BREAK(1);
END;
Y:=Ynew;
END;
CalcA();
CalcV();
CalcT();
CalcDh();
CalcFr();
SolvedY:="** "; //flag Y when printing
END;

//Case 3 Solve slope S given Q,Y,N,T1,Y1
IF SolveNum=3 THEN
CalcC();
CalcA();
CalcP();
CalcR();
S:=(N*Q/(K*A*R^(2/3)))^2;
CalcV();
CalcT();
CalcDh();
CalcFr();
SolvedS:="** "; //flag S when printing
END;

//Case 4 Solve Manning's N given Q,Y,S,T1,Y1
IF SolveNum=4 THEN
CalcC();
CalcA();
CalcP();
CalcR();
N:=(K/Q)*A*R^(2/3)*S^(1/2);
CalcV();
CalcT();
CalcDh();
CalcFr();
SolvedN:="** "; //flag N when printing
END;
END; //end case

//save Y1, T1, UnitsP, and SolveNum to unused system variables for future reference
L:=Y1;
M:=T1;
E:=UnitsP;
F:=SolveNum;

// Print the Results
PRINT();
PRINT("Flow in a Parabolic Channel, y=Cx^2");
PRINT("Channel: C="+ROUND(C,6)+" Y1="+ROUND(Y1,2)+FTm+" T1="+ROUND(T1,2)+FTm);
PRINT(" ");
PRINT(SolvedN+"N = "+ROUND(N,4));
PRINT(SolvedQ+"Flowrate Q = "+ROUND(Q,4)+CFScms);
PRINT(SolvedS+"Slope S = "+ROUND(S,6));
PRINT(SolvedY+"Depth Y = "+ROUND(Y,4)+FTm);
PRINT("Velocity V = "+ROUND(V,4)+FPSmps);
PRINT("Top Width T = "+ROUND(T,4)+FTm);
PRINT("Area A = "+ROUND(A,4)+SFsm);
PRINT("Hydraulic Radius R = "+ROUND(R,4)+FTm);
PRINT("Wetted Perimeter P = "+ROUND(P,4)+FTm);
PRINT("Froude No. = "+ROUND(Fr,4));

END; // main program

//Subroutines

//Calculate the parabola curvature coefficient C from a given top width T
```

```
//and depth Y, where y=Cx^2 is the equation of the parabola.
CalcC()
BEGIN
C:=4*Y1/T1^2;
END;

//Calculate the cross sectional area A, given the depth Y and curvature coef C
CalcA()
BEGIN
A:=4*Y^(3/2)/(3*C^(1/2));
END;

//Calculate the wetted perimeter P, given the depth Y and curvature coef C
//Exact formula by Gary P Merkley, which is equivalent to Ven Te Chow's exact formula
CalcP()
BEGIN
P:=2*C*SQRT(Y/C*((Y/C)+1/(4*C^2))) + 1/(2*C)*ln(2*C*(SQRT(Y/C)+SQRT(Y/C + 1/(4*C^2))));
END;

//Calculate the hydraulic radius R, given the area A and wetted perimeter P
CalcR()
BEGIN
R:=A/P;
END;

//Calculate the velocity V, given the flow rate Q and cross sectional area A
CalcV()
BEGIN
V:=Q/A;
END;

//Calculate the water surface top width T, given the depth Y
//and curvature coefficient C
CalcT()
BEGIN
T:=2*SQRT(Y/C);
END;

//Calculate the hydraulic mean depth Dh, given the cross sectional area A
//and top width T
CalcDh()
BEGIN
Dh:=A/T;
END;

//Calculate the Froude Number, given velocity V, gravitational constant G,
//and hydraulic mean depth Dh
CalcFr()
BEGIN
Fr:=V/SQRT(G*Dh);
END;
```